MATHEMATICS

TEACHER GUIDE
GRADE 9

FEDERAL DEMOCRATIC REPUBLIC OF ETHIOPIA
MINISTRY OF EDUCATION

Price ETB 167.00

Mathematics

Teacher Guide Grade 9

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FEDERAL DEMOCRATIC REPUBLIC OF ETHIOPIA
MINISTRY OF EDUCATION

HAWASSA UNIVERSITY

First Published xxxxx 2022 by the Federal Democratic Republic of Ethiopia, Ministry of Education, under the General Education Quality Improvement Program for Equity (GEQIP-E) supported by the World Bank, UK's Department for International Development/DFID-now merged with the Foreign, Common wealth and Development Office/FCDO, Finland Ministry for Foreign Affairs, the Royal Norwegian Embassy, United Nations Children's Fund/UNICEF), the Global Partnership for Education (GPE), and Danish Ministry of Foreign Affairs, through a Multi Donor Trust Fund.

The Ministry of Education wishes to thank the many individuals, groups and other bodies involved – directly or indirectly – in publishing this Textbook. Special thanks are due to Hawassa University for their huge contribution in the development of this textbook in collaboration with Addis Ababa University, Bahir Dar University, Jimma University and JICA must project.

Printed by:

xxxxxxxx PRINTING

P.O.Box xxxxxx

xxxxxxx, ETHIOPIA

Under Ministry of Education Contract no. xxxxxxxxxxx

ISBN: 978-999944-2-046-9

Foreword

Education and development are closely related endeavors. This is the main reason why it is said that education is the key instrument in Ethiopia's development and social transformation. The fast and globalized world we now live in requires new knowledge, skill and attitude on the part of each individual. It is with this objective in view that the curriculum, which is not only the Blueprint but also a reflection of a country's education system, must be responsive to changing conditions.

It has been almost three decades since Ethiopia launched and implemented new *Education and Training Policy*. Since the 1994 *Education and Training Policy* our country has recorded remarkable progress in terms of access, equity and relevance. Vigorous efforts also have been made, and continue to be made, to improve the quality of education.

To continue this progress, the Ministry of Education has developed a new General Education Curriculum Framework in 2021. The Framework covers all pre-primary, primary, Middle level and secondary level grades and subjects. It aims to reinforce the basic tenets and principles outlined in the *Education and Training Policy*, and provides guidance on the preparation of all subsequent curriculum materials – including this Teacher Guide and the Student Textbook that come with it – to be based on active-learning methods and a competency-based approach.

In the development of this new curriculum, recommendations of the education Road Map studies conducted in 2018 are used as milestones. The new curriculum materials balance the content with students' age, incorporate indigenous knowledge where necessary, use technology for learning and teaching, integrate vocational contents, incorporate the moral education as a subject and incorporate career and technical education as a subject in order to accommodate the diverse needs of learners.

Publication of a new framework, textbooks and teacher guides are by no means the sole solution to improving the quality of education in any country. Continued improvement calls for the efforts of all stakeholders. The teacher's role must become more flexible ranging from lecturer to motivator, guider and facilitator. To assist this, teachers have been given, and will continue to receive, training on the strategies suggested in the Framework and in this teacher guide.

Teachers are urged to read this Guide carefully and to support their students by putting into action the strategies and activities suggested in it.

For systemic reform and continuous improvement in the quality of curriculum materials, the Ministry of Education welcomes comments and suggestions which will enable us to undertake further review and refinement.

ADDIS ABABA, ETHIOPIA *FEDERAL DEMOCRATIC REPUBLIC OF ETHIOPIA*

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Welcoming message to the teacher

Dear teacher, this teacher guide is a curriculum material prepared for you to use with your students. It is a material separately prepared for grade 9 Mathematics teacher. Grade 9 textbook has 8 units namely: The Number System, Solving Equations, Solving Inequalities, Introduction to Trigonometry, Regular Polygons, Congruency and Similarity, Vectors in two dimensions, statistics and Probabilities respectively. Since the students' textbook is basically unitized, you are advised to follow the four components of each lesson and provide the required assistance to the students regularly. Generally, the four components of each lesson are activity, definition/theorem/note, example and exercises. Therefore, you are expected to play your role accordingly.

The components of grade 9 teacher's guide includes:

1. List of general contents and sub-contents of grade 9 students' text book
2. List of general objectives of each unit
3. Suggested teaching aids of each unit
4. Expected students' competencies at the end of each sub-unit
5. Elaborated presentation of each main topic focusing on the following:
    - Some selected topics which are elaborated for you to use as reference material;
    - Identified key ideas to be stressed in each topic;
    - Suggested strategies and sequences of presenting key ideas;
    - Suggestion of alternative methods and techniques for teaching particular topics;
    - Suggested plan and allotted periods of each unit and
    - List of answer keys for each of the activities and exercises of every unit.

Contents

Unit 1

THE NUMBER SYSTEM (33 periods)

Introduction

The main objective of this unit is to survey systematically the numbers we have been dealing with so far and to calculate with them. The purpose is to make the students familiar with the notion of real numbers and calculate with them. As an introduction, you can revise the number systems like integers and rational numbers and their essential properties that were covered in the previous grade levels. The classification of the real numbers as rational and irrational numbers should be clear to the students.

To let them know irrational numbers, locating a point on the number line and trying to represent it as a rational number will be considered. Locating a point on the number line was also dealt with previously. In the previous grades, students have already learned that there are points on the number line to which no rational number can be assigned. In this unit, students will learn about irrational numbers and real numbers. The notion of irrational number as neither repeating nor terminating decimal will also be discussed in this unit. Further representation of subsets of a real number using intervals will be studied. The correspondence between number and point on a number line should be stressed.

In addition to these, students should study how the concept of the square root of a number such as $\sqrt{2}$ leads to the definition of an irrational number. In relation to this, the concept of radicals, the notion of rationalization and its use in simplifying expressions involving radicals should be covered. After dealing with the number systems, some related concepts such as approximation, accuracy and scientific notation will be discussed. Finally the application of some concepts in this unit like LCM, GCF and Euclid's division algorithm will be presented.

Unit Outcomes

After completing this unit, students will be able to:-

- describe rational numbers
- locate rational numbers on number line
- describe irrational numbers

- locate some irrational numbers on a number line
- define real numbers
- classify real numbers as rational and irrational
- solve mathematical problems involving real numbers.

Suggested Teaching Aids in Unit 1

The teaching aids availability may vary from school to school. For this unit, you can prepare and present different charts that manifest squares of whole numbers and multiplication tables. For teaching irrational numbers, you need a pair of compass and ruler to locate irrational numbers such as $\sqrt{2}$, $\sqrt{3}$ etc. on the number line. You also need scientific calculators. If possible and your school has a computer lab, take some time for students to show these irrational numbers using mathematical software like GeoGebra. But do not allow students to use scientific calculators frequently even for simple calculations.

1.1. Revision on Natural Numbers and Integers

Period allotted: 7 periods

Competencies

At the end of this subunit, the students will be able to:

- identify natural numbers and integers
- define Euclid's division algorithm
- apply Euclid's division algorithm to solve word problem
- test divisibility of numbers
- define prime numbers and composite numbers
- determine greatest common factors and least common multiples of numbers

Introduction

This sub-unit deals with revising the set of integers together with their important properties. However, this is done first by discussing the set of natural numbers, prime numbers, composite numbers. Related to these numbers, the concepts of factors, multiples, prime factorization, common factors, common multiples, greatest common factor and least common multiple are discussed. For each concept, an activity, motivating questions and group work are provided to refresh the memory of the students or to guide them to the concepts.

Teaching Notes

When you begin the lesson, inform students where the available supporting materials are located, when they can be accessed and aware the weekly counseling hours you arranged. It is also better to inform students what learning material is needed. Motivate the students by giving an insight of the course and the units. You can also highlight the subtopics of this unit. Following these discussions, you can continue to discuss the subsections.

You can begin by stating the objective of the lesson and go on revising the number system. To do this, you may use Activity 1.1 for the purpose of revising student's prior knowledge on the various number systems. Group your students and let them discuss Activity 1.1. Give time to discuss in group, let some of the groups' representative present their discussion to the whole class. You can then facilitate their discussion. This will again lead you to discuss the definitions of natural numbers and integers.

Possible Teaching aid : Diagrams which show the relationship between number system

Revision on natural numbers and integers

Period allotted:-1 period

The lesson focuses on reminding what are natural numbers and integers. You can motivate students by raising a question like' what would happen if there is no concept of numbers?', or when do human being start to use numbers. Students may reflect their feeling about the use of numbers and you can facilitate the oral discussion. After this you can give activity 1.1 and give the feedback after their trial.

Answer for activity 1.1

1. **a.** There are infinitely many solutions for this. One of the solutions is $5,7,8,40$ and 100

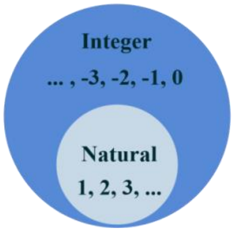
   b. There are infinitely many solutions for this. One of the solutions is $-4,-1,0,3$ and 500
2. 6,0 and -25 are integers whereas 6 is a natural number
3. The set of natural numbers are included in integers
4. **a.** always true **b.** sometimes true **c.** never true
5. 

Figure 1.1

Answer for Exercise 1.1

1. $8, 23, 534, 100$ are natural numbers and $8, -11, 23, 534, 0, -46, -19, 100$ are integers

2. 999

3. **a.** For any two natural numbers n_1 and n_2, their sum is also a natural number. For instance take 2 and $9, 2 + 9 = 11$ is also a natural number.

b. For any two natural numbers n_1 and n_2, their difference can be natural number if $n_1 > n_2$ like $8 > 5, 8 - 5 = 3$ is a natural number and their difference is not a natural number if $n_1 < n_2$, like $1 < 12$, and $1 - 12 = -11$ is not a natural number.

c. From the above two cases we conclude that the set of natural numbers is closed under addition and it is not closed under subtraction.

4. We need to find the possible value of sides of the triangle whose perimeter is 10.

Let us consider triangle ABC whose side lengths are $AB = a, BC = b$ and $AC = c$. As we know the property of a triangle is ''**the sum of two sides of a triangle is greater than the third side**''

Therefore, $AB + BC > AC$ so that $AB + BC > 5$.

The possible combinations for sides of the triangle are $(2,4,4)$ and $(3,3,4)$.

5. Using the first criteria, let us list the possible numbers, suppose (m, n) be the list which satisfy the criteria, $m + n < 10$ and different value of mn

$(1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8)$

$(2,1), (2,3), (2,4), (2,6), (2,7)$

$(3,1), (3,2), (3,4), (3,5), (3,6)$

$(4,1), (4,2), (4,3), (4,5)$

$(5,1), (5,2), (5,3), (5,4)$

$(6,1), (6,2), (6,3)$

$(7,1), (7,2)$

$(8,1)$

Then remove the one which gives the same product like $(1,3)$ and $(3,1)$. So that Remove the repeated one. For this Purpose, highlighted the repeated one from the list by red colour and count the remaining terms, so that we have 16 possibilities

1.1.1 Euclid's division lemma

Period allotted:-1 period

Whenever two natural numbers are given, we will try to experience division of the first by the second or vice versa. This could be done by a long division process. At this subtopic, once the dividend is less than the divisor, we stop the process and the number remained is remainder. So before writing the division algorithm, give activity 1.2 to the students as a group work to recall their understanding about division of a number by another number. Then write the theorem and give illustrative examples. Finally give the exercise.

Answer for Activity 1.2

1. This is to mean, divide 115 by 8 and determine the quotient and remainder.

If we divide 115 by 8 we got 14 and the remaining 3.

So to give interpretation for the word problem, each student received 14 books and 3 books are left.

2. When we divide 128 by 6, we get 21 as a quotient and its remainder is 2. The remainder should be a non-negative integer less than 6.

Answer for Exercise 1.2

1. **a.** $14 = 3 \times 4 + 2$, quotient 4 and remainder 2.

b. $116 = 7 \times 16 + 4$, quotient 16 and remainder 4.

c. $25 = 36 \times 0 + 25$, quotient 0 and remainder 25.

d. $987 = 16 \times 61 + 11$, quotient 61 and remainder 11.

e. $570 = 6 \times 95 + 0$, quotient 95 and remainder 0.

2. Using division algorithm let a is dividend, b is divisor, q is quotient and r be remainder we have $a = bq + r$ where $0 \leq r < b$.

Given that $b = 4$ and $r = 3$, now we need to find a using division lemma.

- To get the first number, let $q = 0$, $b = 4$ and $r = 3$ (both are given)

Therefore $a = 4(0) + 3 = 3$.

- To get the second number let $q = 1$, $b = 4$ and $r = 3$ (both are given)

Therefore $a = 4(1) + 3 = 7$.

- To get the 3rd number let $q = 2$, $b = 4$ and $r = 3$ (both are given)

Therefore $a = 4(2) + 3 = 11$.

Hence, the next number will be 4 more than the preceding number.

Therefore, the possible positive integers are $3,7,11,15, \ldots$

3. The man plan to buy items that is the dividend is $a = 68$ Birr and each item price is $b = 7$ Birr. He needs $r = 5$ Birr to remain. Now what we need is the maximum number of items with the above condition. By applying Euclid's lemma, $a = bq + r$.

$68 = 7q + 5$, solving for q we get $q = 9$.

This means he can buy 9 items each with a price of 7 Birr.

1.1.2 Prime numbers and composite numbers

Period allotted:-1 period

In this subsection, you will help students to recall important facts about prime and composite numbers. They have learnt about these facts in the previous grades. Group the students and ask them to do activity 1.3. Let some of the groups present their work to the class. Then start discussing the answer to each question with the students. Make sure that students understand the definitions and concepts given in the lesson; in particular, make sure that they can distinguish between the set of natural numbers, and integers, prime numbers and composite numbers. Some students confuse prime numbers with odd numbers. Here, you are expected to make sure that students are able to distinguish between a prime number and an odd number.

Possible Teaching aid: Material needed (Hard paper, different colored markers) **.** You can list natural numbers (1-50) horizontally on a hard paper. Underline prime numbers with one color and composite numbers with another color.

Answer for Activity 1.3

1. **a.** true
b. false , since there exist 2 which is an even prime number
c. true
d. false , 1 is neither prime nor composite.
e. true
f. true
g. true

2. **a.** The factors of 7 are 1 and 7. **b.** The factors of 15 are 1,3,5 and 15

3. **a.** 3 ,12 **b.** 12,3 **c.** 12,3

After defining the prime and composite number, group activity need to be given so as to provide a chance to students to revise what they have learnt from in lower grades. Let some of the groups present their work to the class. Then start discussing the answer to each question with the students. Make sure that students understand the basic concept which is prime factorization.

Answer for Exercise 1.3

1. $2, 3, 5, 7, 11, 13, 17, 19, 23, 29$
2. **a.** False **d.** True
   b. True **e.** True
   c. False
3. 28 could be written as $\left.\begin{array}{r} 28 = 1 \times 28 \\ 28 = 2 \times 14 \\ 28 = 4 \times 7 \end{array}\right\}$. The factors of 28 are 1,2,4,7,14 and 28.
4. No, there are many odd numbers which are composite. For instance 9,25,27, ...
5. D

1.1.3 Divisibility test

Period allotted:-1 period

Now the students have learned how to divide a natural number by another natural number by using Euclid's algorithm. But this may take time. So enforce students to identify whether a certain number is divisible by another one or not. For instance if the unit digit of a number is even it is divisible by 2. You can first give activity 1.4 to see how they approach the problem and help to remember what they have learned in lower grades. Finally give those divisibility test and support with examples. When you jump divisibility by 7, a student may raise a question. In that case inform the student that it needs advanced concepts so that it will not be treated at this level. Finally give exercise 1.4 as homework.

Possible Teaching aid: You can write the divisibility test written on page 7 of the textbook on a hard paper with large size and hung on the wall.

Answer for Activity 1.4

Here we expect yes it is possible to dividing the first by the second or not.

a. yes **b.** yes **c.** no **d.** yes **e.** yes

Using the drivability test, proceed to write a number as a product of prime numbers and discuss the fundamental theorem of arithmetic. Give different examples to address the understanding of the concepts. In the middle of your discussion ask an oral question. For instance you may write a number 651 and ask students whether it is divisible by 3 and ask them why? This may help student to be active in the lesson. Use colored chalk while you factorize a number by factor tree.

Answer for Exercise 1.4

1.

   a. 384 is divisible by 2,3,4,6 and 8 but not divisible by 5,9 and 10.

   b. 3,186 is divisible by 2,3,6 and 9 but not by 4,5,8 and 10.

   c. 42,435 is divisible by 3 ,5and 9 but not by 2,4,6,8 and 10.

2. The number $74{,}3x2$ is divisible by 4 if its last two digit is divisible by 4, hence it will happen when x is 1,3,5,7 and 9.

3. Divisibility by 4 depends on the divisibility of the last two digits by 4. Here 40 divisble by 4. Hence the given number is divisible by 4 if we replace the blank space by any digit. The minimum is zero. So that the least number we use in the blank space is 0.

4. In order to be divisible by 9, the sum of the digits should be divisible by 9. Here we have $8+1+2+3+1+3+7=25$ hence if we add 2 this sum will be divisible by 9. Hence the minimum digit we use in the blank space is 2.

Prime factorization

Period allotted:-1 period

This exercise could be taken as classwork

Answer for Exercise 1.5

1. Students can use factor tree to determine the prime factorization of the numbers

   a. 3×7 **b.** $2\times 5\times 7$ **c.** $3\times 5\times 7$ **d.** $2^2\times 3^2\times 7$ **e.** $2^3\times 3^2\times 5$

2. This is another way of asking find the prime factorization of 180. Using factor tree we get that $180=2\times 2\times 3\times 3\times 5=2^2\times 3^2\times 5^1$. Hence, $a=2, b=2$ and $c=1$.

This exercise could be given as homework

Answer for Exercise 1.6

1. a. $1{,}848\div 2=924$

$924\div 2=462$

$462 \div 2 = 231$

$57 \div 3 = 77$

$77 \div 7 = 11$. This implies that $1{,}848 = 2^3 \times 3 \times 7 \times 11$

b. Following similar procedure as that of (a) $1{,}890 = 2 \times 3^3 \times 5 \times 7$.

c. The prime factorization of $2{,}070$ will be

$2070 \div 2 = 1035$

$1035 \div 3 = 345$

$345 \div 3 = 115$

$115 \div 5 = 23$

23 is a prime number. So that$2{,}070 = 2 \times 3^2 \times 5 \times 23$.

d. $34{,}750 = 2 \times 5^3 \times 7^2 \times 11$.

2. 2 divides 462, so that

$462 \div 2 = 231$

$231 \div 3 = 77$

$77 \div 7 = 11$

This implies that $2 \times 3 \times 7 \times 11 = 462$. The required prime factors are $2{,}3{,}7$ and 11.

Assessment

You can assess the students whether they understood the divisibility test by giving some more exercise similar to Exercise 1.6 which could be taken as an assignment.

1.1.4 Greatest common factor and least common multiple

Period allotted:-2 periods

Students have studied about the Greatest Common Factor (GCF) or Highest Common Factor (HCF) and the Least Common Multiple (LCM) of two natural numbers. So you can start the session by asking students, if they can remember what they learned about GCF and LCM. After getting answers from two or three students give time to practice activity 1.3.

While students practice to find the GCF and LCM of numbers, you will need to check each student tries his or her work; whenever he/she has no any idea, give some hint to list multiples and factors of a number.

Finally, give a chance to some students to reflect on the procedure they follow and invite students to ask any question about the lesson.

Answer for Activity 1.5

1. Given the numbers 12 and 16

a. $1,2,3,4,6,12$ are factors of 12 and $1,2,4,8,16$ are factors of 16. So that

Then select the common factors of the two numbers. 1,2,4 are common factors .

b. The largest of this set is 4, so that the greatest common factor is 4.

2. In similar way to case (a)

a. The common factors are 1 and 2.

b. The greatest number of the common factors is 2. Hence, 2 is the greatest common factor of $24, 42$ and 56.

After this activity define the common factors and greatest common factorization (GCF). In some books it is also written as the highest common factor (HCF). So inform the students about these two naming since they may get these two if they refer to different reference materials. Then elaborate these ideas by giving two or more examples first for two numbers and then for three natural numbers. In your discussion, engage students by asking questions about how to check whether if the set of common factor is correct or not.

For example: $1,2,3,6,9,18$ are factors of 18 since, 1×18, 2×9, 3×6 all these give the number 18. But they may miss some factors so tell them to use a divisibility test while they determine factors.

Give Activity 1.6, in a group of 3-4 members and follow up each student's participation. Remind students how to write numbers as a product of prime numbers and finally give chance for group members (randomly) to reflect the group result.

Answer for Activity 1.6

1. The prime factorization of the two numbers $a = 36$ *and* $b = 56$ are $a = 2^2 \times 3^2$ and $b = 2^3 \times 7$.

2. The prime number which is common to the two numbers is 2

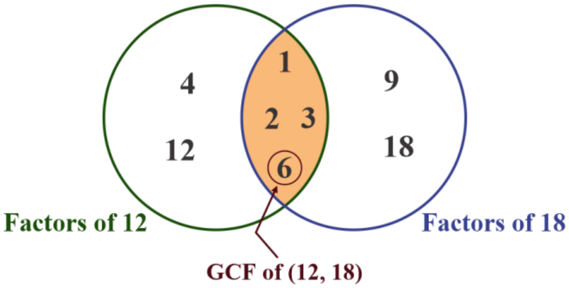
3. Since we have only one common factor 2 with least power 2, so that the GCF(36,56)=4

4. In the example 1 page 12 of students textbook , we have the same result by Venn diagram.

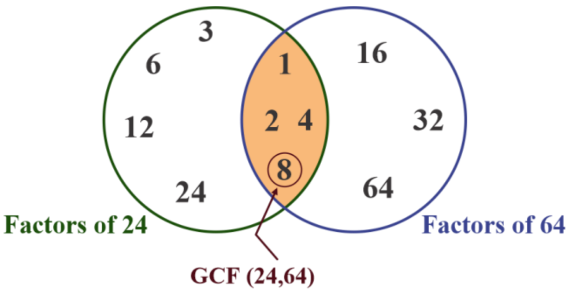
Answer for Exercise 1.7

1. Using Venn diagram

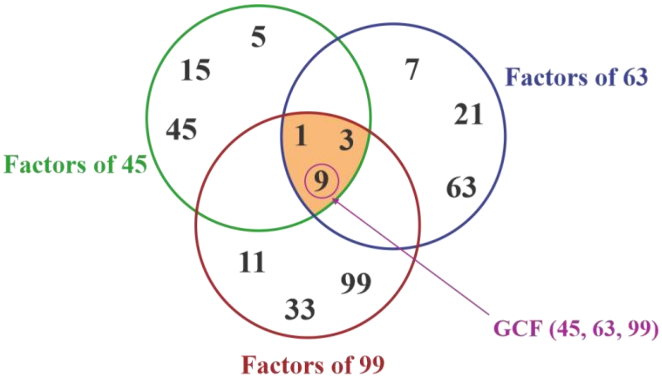
   a. Factors of 12 are 1,2,3,4,6 and 12 and factors of 18 are 1,2,3,6,9 and 18. The common factors are 1,2,3 and 6. Hence 6 is the greatest common factor of 12 and 18 .



   b. Similar to (a) the GCF (24,64) is 8



   c. GCF(45,63,99) is 9 as sown in the following Venn diagram



2. Using prime factorization

   a. $24 = 2^3 \times 3$

   $54 = 2 \times 3^3$

   $\mathrm{GCF}(24,54) = 2 \times 3 = 6$

   b. $108 = 2^2 \times 3^3$

   $104 = 2^3 \times 13$

   $\mathrm{GCF}(108,104) = 2^2 = 4$

c. $180 = 2^2 \times 3^2 \times 5$

$270 = 2 \times 3^3 \times 5$

$1{,}080 = 2^3 \times 3^3 \times 5$

$\text{GCF}(180{,}270{,}1{,}080) = 2 \times 3^2 \times 5 = 90.$

For those students who have difficulty on identifying give a activity 1.7 for the learners. In this activity learners should have pencil and pen or colored pen to identify the intersection of the two multiples.

Answer for Activity 1.7

We will list natural numbers from 1 to 60 as follows

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60

a. 24,48

b. 24 ,this number 24 is called the least common multiple

Then define the Least Common Multiple (LCM) of natural numbers and give examples to illustrate the definition.

Answer for Exercise 1.8

a. $6 = 2 \times 3$

$15 = 3 \times 5$

$\text{LCM}(6{,}15) = 2 \times 3 \times 5 = 30$

b. $14 = 2 \times 7$

$21 = 3 \times 7$

$\text{LCM}(14{,}21) = 2 \times 3 \times 7 = 42$

c. $4 = 2^2$

$15 = 3 \times 5$

$21 = 3 \times 7$

$\text{LCM}(4{,}15{,}21) = 2^2 \times 3 \times 5 \times 7 = 420$

d. $6 = 2 \times 3$

$10 = 2 \times 5$

$15 = 3 \times 5$

$18 = 2 \times 3^2$

$\text{LCM}(6,10,15,18) = 2 \times 3^2 \times 5 = 90$

Answer for Activity 1.8

Activity is given to the learner in order to provide an alternative method of determining LCM using the factorization method and to see the relationship between GCF and LCM.

a. $15 = 3 \times 5$ and $42 = 2 \times 3 \times 7$, then $\text{GCF}(15,42) = 3$ and

$\text{LCM}(15,42)=2 \times 3 \times 5 \times 7 = 210$

b. $15 \times 42 = 630$

c. 630

d. They are the same

e. $\text{GCF}(a, b) \times \text{LCM}(a, b) = ab$ for any two natural numbers a and b.

After this class activity help the learners to generalize that $\text{GCF}(a, b) \times \text{LCM}(a, b) = ab$ for any two natural numbers a and b. Be sure by asking them a question like 'Can you get GCF of two numbers if LCM of the numbers is given? ' . Then give Exercise 1.9.

Answer for Exercise 1.9

The learner can approach the problem by whatever method they need

1.

a. $\text{GCF}(4,9) = 1$ (they are called relatively prime numbers). You can also help students to have an observation, if two numbers are relatively prime; their product is the same as their LCM.

$\text{LCM}(4,9) = 36$

b. $\text{GCF}(7,48) = 1, \text{LCM}(7,48) = 336.$

c. $\text{GCF}(\ 12,32)\ = 4, \text{LCM}(\ 12,32) = 96.$

d. $\text{GCF}(16,39) = 1, \text{LCM}(16,39) = 624.$

e. $\text{GCF}(12,16,24) = 4, \text{LCM}(12,16,24) = 48.$

f. $\text{GCF}(4,18,30) = 2, \text{LCM}(4,18,30) = 180$

2. $GCF(16,24) \times LCM(16,24) = 384$

Assessment

Write different positive integers (10-12) on the chalk board (even, odd and composite) . Pick a student randomly to circle any two of them on the blackboard. Ask another student to find GCF and LCM of the two numbers. Do a similar task and check whether they have understood how to find LCM and GCF of two or more numbers.

1.2 Rational Numbers

Period allotted:- 4 periods

You can start the lesson by asking the students which operations are closed in the set of integers. You can write integers like 5 and 9, ask them to add, subtract, divide and multiply these numbers. Getting their reply will lead the student to understand the set of integer is not closed under division since $5 \div 9$ is not an integer is closed under division. Hence, such an approach will enforce to find the biggest set which may fill the above types of numbers.

Competencies

At the end of this subsection a student will be able to :-

- describe rational numbers
- locate rational numbers on the number line
- show that repeating decimals are also rational numbers
- convert decimals to fractions and vice versa.

Before defining the set of rational numbers give time to students to do activity 1.9.

Answers for Activity 1.9

a. $\frac{7}{7}, \frac{7}{-2}, \frac{7}{6}, \frac{7}{-3}, \frac{-2}{7}, \frac{-2}{-2}, \frac{-2}{6}, \frac{-2}{-3}, \frac{6}{6}, \frac{6}{7}, \frac{6}{-2}, \frac{0}{7}, \frac{-3}{-3}, \frac{-3}{-2}, \frac{-3}{6}$

b. $\frac{7}{7}, \frac{-2}{-2}, \frac{6}{6}, \frac{6}{-2}, \frac{0}{7}, \frac{-3}{-3}$

c. If we divide one integer by another the result is not necessarily integer.

Then discuss the set of rational numbers formally by elaborating with the help of examples. Give stress that in $\frac{a}{b}$,$b \neq 0$. Furthermore, ask students to mention some situations in which, people are using rational numbers in their day to day activities. For example, one injera is divided to four individuals equally, each individual gets one forth $(\frac{1}{4})$ of the full injera. Another approach may be if you want a distance using the unit meter, you may have a measurement which is less than 1 meter, it may be 50cm or 25cm and so on. If you change these into meter, the corresponding measurement in meters will be ½ meter for 50cm and ¼ meter for 25cm. These numbers ½ or ¼ are rational numbers. Remind also the numbers in the set of integers are rational numbers since all these numbers have a denominator 1 and it satisfies the definition of rational number.

Rational numbers

Period allowed: 1 period

Answer for Exercise 1.10

1.

- **a.** True
- **b.** False (it is 7/9)
- **c.** False (those restrictions $b \neq 0, d \neq 0$ should be mentioned)
- **d.** True

2. Since $\frac{a}{b}$ and $\frac{c}{d}$ are both rational number, by definition both b and d are different from zero. Now take the product of the two numbers $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ this number is a rational since the denominator $b \times d \neq 0$.Furthermore, $a \times c, b \times d$ should be integers.

3. 4mm is 4/10cm, so that the length of a table will be $54 + \frac{4}{10} = 54 + \frac{2}{5} = \frac{272}{5}$cm.

4. The length of each piece is calculating $\frac{5\frac{1}{3}}{4} = \frac{\left(\frac{16}{3}\right)}{4} = \frac{4}{3}$ meters.

1.2.1. Representation of rational numbers by decimals

Period allowed: 1 period

On the definition of a rational number, a fraction form $\frac{a}{b}$ is given. But using division, that divides a by b, you will get a quotient and a reminder. This is to mean changing in to decimal form. In the process of division you may stop by getting zero remainder or repeated remainder. The first type is a decimal which is a terminated decimal and the second type is a repeating decimal. For instance using division $\frac{1}{2} = 0.5$ and $\frac{1}{3} = 0.\overline{3}$.

Every rational number can be located on the number line. But students can visualize its location if you write it in decimal form. So give activity 1.6 to the students to see their understanding about how to change the fraction to decimal form.

Possible teaching aids: Ruler, coloured chalk, hard paper, and marker.

Answers for Activity 1.10

1. **a.** 0.6 **b.** $0.555\ldots$ **c.** $-0.9\overline{3}$ **d.** $-0.\overline{285714}$

2. $0.2 = \frac{1}{5}$, $3.31 = \frac{331}{100}$

After doing their activity, discuss the process of changing fractions to rational and vice versa. Give also some more examples till you are sure about student understands the terminating and repeating decimals. Finally give exercise 1.11 and some more problems as homework. Do not encourage the students to use calculator since it may lead the student to be machine dependent even for simple calculation.

Answers for Exercise 1.11

1. **a.** $0.\overline{8}$ **b.** $0.8\overline{3}$ **c.** $0.\overline{09}$ **d.** $1.\overline{5}$

2. Yes we can , $\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$,

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571},$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428},$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285},$$

$$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}.$$

3. If we multiply a repeating decimal by a natural number, we obtain repeating decimal.

Answers for Exercise 1.12

1. **i) a.** $\frac{3}{10}$ **b.** $\frac{37}{10}$ **c.** $\frac{77}{100}$ **d.** $-\frac{12369}{1000}$

a. $\frac{6}{10} = \frac{3}{5}$ **b.** $\frac{95}{10} = \frac{19}{2}$ **c.** $\frac{-48}{100} = \frac{-24}{50} = \frac{-12}{25}$ **d.** $\frac{-32125}{1000} = \frac{-257}{8}$

Representing rational numbers on the number line

Period allowed: 1 period

For the topic of rational number representation on the number line, you may start the lesson by asking a question like can you locate the following rational numbers $-4, \frac{1}{5}, -2, \frac{20}{6}$ on the numbers line? Invite interested students to come to the chalkboard and locate the given numbers on the number line. For lessons on repeating decimals help the students to enlarge the interval they use and also coloured chalk.

Finally to check the students' level of understanding regarding this concept, assign them to do Exercise 1.13.

Answer for Exercise 1.13

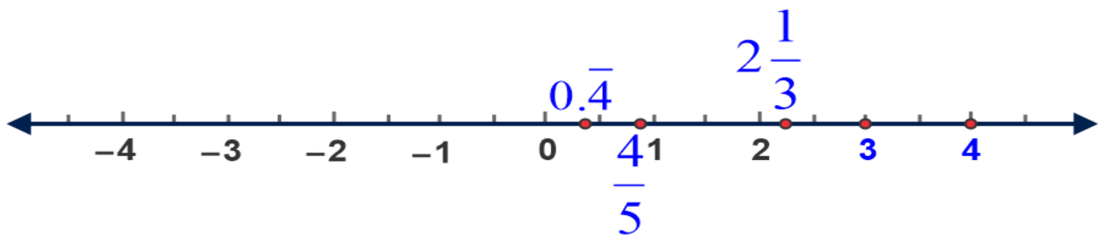


Figure 1.2

1.2.2 Conversion of repeating decimals into fractions

Period allowed: 1 period

Students obtain the decimal representation of a rational number from fractions to decimals, simply by using long division. In the previous sub topic students practice how to convert terminating decimals to fractions. In this subtopic you will focus on how to convert repeating decimals to fractions.

You can follow the examples given in the students' text book to approach this section. You can also add some more additional examples to clarify the concept step by step, first a decimal with one repeating digit, then two and three repeating digits. To write in simplest form helps the students to recall the divisibility test. From exercise 1.6, give two of them as a class work.

Answers for Exercise 1.14

a. Let $a = 2.\bar{6}$ so that $10a = 26.\bar{6}$. It follows that $9a = 24$ n simplest form $a = \frac{8}{3}$.

b. In similar way as that of a (a) $0.\overline{14} = \frac{14}{99}$.

c. Let d be the given decimal , that is $d = 0.7\overline{16}$, multiply d by 1000 and subtract $10d$ from $1000d$ we obtain $990d = 709$, so that $d = \frac{709}{990}$.

d. Let $= 1.32\overline{12}$, $100a = 132.\overline{12}$ and $10000a = 13212.\overline{12}$. $9900a = 13080$

The simplified answer is $\frac{218}{165}$.

e. Using the above formula we obtain $\frac{-52681}{99000}$

Assessments

You can assess your students by giving them various exercises of converting fractions form into decimals and decimals into fractions. You can let students do these as homework and present their work.

1.3 Irrational Numbers

Period allotted: 5 periods

The main task of this subunit is to make students familiar with the notion of irrational numbers. It is not a matter of extending rational numbers to irrational numbers. This is a different category which consists of number that is not rational. After defining irrational numbers, represent these numbers on the number line. Finally, there will be discussion about operations on irrational numbers.

Competencies

At the end of this section a student will be able to the students

- identify irrational numbers
- locate some irrational numbers on a number line
- perform any one of the four operations on the set of irrational numbers.

1.3.1 Neither repeating nor terminating numbers

Irrational number (1)

Period allowed: 1 period

Start the lesson by asking students ''Have you learned perfect squares?'' If so, give them time to think and get two or three students reply. Then list some perfect squares and inform them why we call it perfect. Tell the students about the notation of square root. Take few examples to practice the meaning of square root. Then ask students ''can you able to get an integer whose square gives 2?'' Help them to test by taking numbers. From the activity, you also help them to locate irrational number $\sqrt{2}$ on the number line. Then discuss the lesson by defining irrational numbers.

Possible teaching aids: Ruler, compass, coloured chalk, hard paper, scientific calculator and marker.

Answer for Exercise 1.15

a. 5 **b.** -6 **c.** 0.2 **d.** -0.09

Answer for Exercise 1.16

a. $\sqrt{7} < \sqrt{8}$

b. $\sqrt{3} < \sqrt{9} = 3$

c. $\sqrt{0.01} < \sqrt{0.04}$

d. $-\sqrt{4} < -\sqrt{3}$ since -2 is located at the left of $-\sqrt{3}$

Irrational number (2)

Period allowed: 1 period

Answer for Exercise 1.17

a. Rational **b.** Irrational **c.** Rational **d.** Rational

e. Irrational **f.** Irrational **g.** Irrational

Locating irrational number on the number line

Period allowed: 1 period

Answer for Exercise 1.18

1. **a.** $\sqrt{3}$ is located between 1 and 2

b. $\sqrt{5}$ is between 2 and 3

c. Also $\sqrt{6}$ is between 2 and 3.

2. **a.** Example 2, students have learnt how to represent $\sqrt{2}$ on the number line. Then follow the following procedure to locate $\sqrt{3}$ on the number line.

-fix a point $\sqrt{2}$ unit long on the number line

-construct a perpendicular line segment at this point which is 1 units long

- a right angled triangle is formed with hypotenuse $\sqrt{3}$ units long (that is $c = \sqrt{3}$)

- open the compass to the length of c. With the tip of the compass at the point corresponding to 0, draw an arc that intersects the number line. This point is $\sqrt{3}$ units long as shown in figure 1.3.

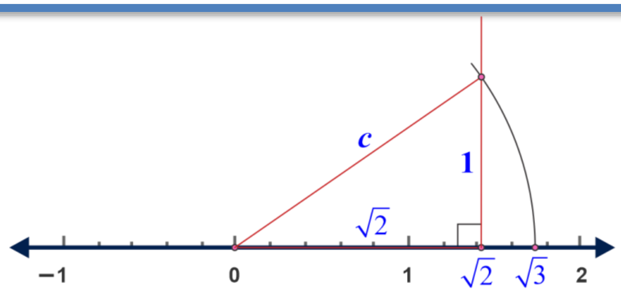


Figure 1.3

b . To locate $\sqrt{5}$ on the number line follow the following procedure

-fix a point say A which is 2 units long on the number line

-construct a perpendicular line segment through A that is 1 unit long

-from a right-angled triangleOAB, $OB = \sqrt{5}$ units long (using Pythagoras theorem)

- open the compass to the length of OB. With the tip of the compass at the point corresponding to 0, draw an arc that intersects the number line. This point is point C, which is $\sqrt{5}$ units long as shown in figure 1.4.

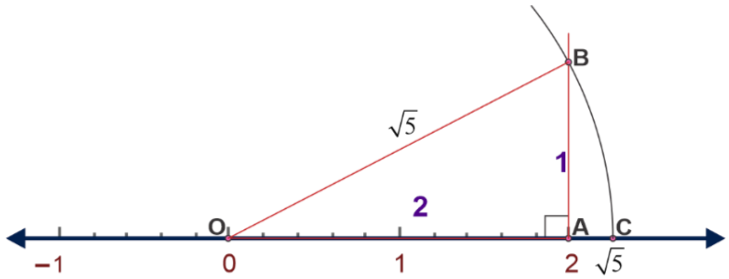


Figure 1.4

c. The location of $-\sqrt{3}$ will be done exactly the same as (a) but the only difference is it is located at the left of the origin

3. a. False (as negative numbers are not expressed as a square root of any other number)

b. False (take for instance $\sqrt{9} = 3$ which is rational)

Assessments

Here you can assess students through several approaches. You can give the students irrational numbers and ask them to locate each on the number line. You can also ask them to describe the properties like closure on the set of irrational numbers. You may also ask them to identify rational and irrational numbers.

1.3.2 Operations on irrational numbers

Period allowed: 2 periods

After students able to locate irrational numbers on the number line, in this subtopic, you should discuss the operations on irrational numbers. You will approach the discussion by giving the activity to the students and reach to a conclusion. That is the set of irrational numbers is not closed under addition, subtraction, multiplication, and division.

Answers for Activity 1.11

1. We know that $\sqrt{2} = 1.4142\ldots$ is an irrational number, so that $1 + \sqrt{2} = 2.14142\ldots$ which is neither terminating nor repeating, so that it is an irrational number.
2. Students can take different irrational numbers, but it will be either

   Irrational × Irrational =Rational , eg. $\sqrt{2} \times \sqrt{2} = 2$ or

   Irrational × Irrational =Irrational , eg. $\sqrt{2} \times \sqrt{3} = \sqrt{6}$.

Answer for Exercise 1.19

1.

a. $\sqrt{15}$ **b.** $2\sqrt{5} \times \sqrt{7} = 2\sqrt{35}$ **c.** $-\sqrt{12}$ **d.** $\frac{1}{\sqrt{5}} \times \frac{10}{\sqrt{5}} = \frac{10}{5} = 2$

e. $(2 + \sqrt{3}) \times (-2 + \sqrt{3}) = -4 + 2\sqrt{3} - 2\sqrt{3} + 3 = -1$

f. $(\sqrt{3} + \sqrt{2})^2 = 3 + 2\sqrt{6} + 2 = 5 + 2\sqrt{6}$ (use $(a + b)^2 = a^2 + 2ab + b^2$)

g. $(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3}) = 7 - 3 = 4$

h. $(\sqrt{6} - \sqrt{10})^2 = 6 - 2\sqrt{60} + 10 = 16 - 2\sqrt{60}$

2.

a. Sometimes true eg. Take 1.e of the above

b. Sometimes true eg. $\sqrt{2} \times \sqrt{3} = \sqrt{6}$

c. Sometimes true , eg. $\sqrt{3} \times \sqrt{5} = \sqrt{15}$

d. Never true eg. $\frac{1}{\sqrt{3}} \times \frac{2}{\sqrt{3}} = \frac{2}{3}$.

Answers for Activity 1.12

1. $\frac{\sqrt{2}}{\sqrt{2}} = 1, \frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}}, \frac{\sqrt{5}}{\sqrt{2}} = \sqrt{\frac{5}{2}},$ and $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = 2$
2. If we divide irrational number an irrational number we can get rational/irrational number.

Answer for Exercise 1.20

1.

a. $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{27}{3}} = \sqrt{9} = 3$

b. $\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$

c. $\frac{\sqrt{14}}{\sqrt{7}} = \sqrt{\frac{14}{7}} = \sqrt{2}$

d. $\frac{-\sqrt{15}}{\sqrt{3}} = -\sqrt{\frac{15}{3}} = -\sqrt{5}$

2.

a. $4\sqrt{2} = \sqrt{4^2 \times 2} = \sqrt{32}$

b. $-3\sqrt{7} = -\sqrt{3^2 \times 7} = -\sqrt{63}$

c. $5\sqrt{3} = \sqrt{5^2 \times 3} = \sqrt{75}$

d. $7\sqrt{6} = \sqrt{7^2 \times 6} = \sqrt{294}$

Answers for Activity 1.13

There might be many option, one of them can be the following

a. $3.2121121112 \ldots + 1.121221222122221\ldots = 4.3333 \ldots$

b. $0.12112111 \ldots + 5.020020002.. = 5.141141114 \ldots$

c. $\sqrt{3} - \sqrt{3} = 0$

d. $\sqrt{5} - \sqrt{3}$

Answer for Exercise 1.21

a. $\sqrt{3} + 4\sqrt{3} = (1+4)\sqrt{3} = 5\sqrt{3}$

b. $\sqrt{5} - \sqrt{45} = \sqrt{5} - 3\sqrt{5} = -2\sqrt{5}$

c. $2\sqrt{5} - 4\sqrt{5} = (2-4)\sqrt{5} = -2\sqrt{5}$

d. $\sqrt{18} + 2\sqrt{2} = 3\sqrt{2} + 2\sqrt{2} = (3+2)\sqrt{2} = 5\sqrt{2}$

e. $0.12345 \ldots - 0.111 \ldots = 0.0123456 \ldots$

f. $\sqrt{80} - \sqrt{20} = 2\sqrt{20} - \sqrt{20} = \sqrt{20} = 2\sqrt{5}$

g. $5\sqrt{8} + 6\sqrt{32} = 5\sqrt{8} + 6 \times 2\sqrt{8} = 17\sqrt{8} = 34\sqrt{2}$

h. $\sqrt{8} + \sqrt{72} = \sqrt{8} + 3\sqrt{8} = 4\sqrt{8} = 8\sqrt{2}$

i. $\sqrt{12} - \sqrt{48} + \sqrt{\frac{3}{4}} = 2\sqrt{3} - 4\sqrt{3} + \frac{\sqrt{3}}{2} = \left(2 - 4 + \frac{1}{2}\right)\sqrt{3} = -\frac{3}{2}\sqrt{3}$

j. $2.1010010001 \ldots + 1.0101101110 \ldots = 3.111 \ldots$

1.4. The Real Number

Period allotted : 16 periods

Competencies

At the end of this subunit, the students will be able to:-

- define real numbers
- describe the correspondence between real numbers and points on a numbers line
- determine the distance of two real numbers of the form $\pm a, a \in \mathbb{R}$, from 0 on a number line, and relate this to the absolute value of a ($|a|$)
- determine the absolute value of a positive or negative real number
- explain, using examples, how distance between two points on a number line can be expressed in terms of absolute value
- determine the absolute value of a numerical expression
- perform any one of the four operations on the set of real numbers
- use the laws of exponents to simplify expression
- compare and order the absolute values of real numbers in a set
- give appropriate upper and lower bounds for a given data to a specified accuracy (e.g. rounding off).
- express any positive rational number in its standard form
- explain the notion of rationalization.

Introduction

Period allowed: 1 period

The main objective of this sub-unit is to make students familiar with the notion of real numbers and their properties systematically. In the previous two sub units students have got the basics on rational and irrational numbers and their properties. The two rational and irrational number systems are not included one in the other. Remind students on the representation of both rational and irrational numbers on the number line. That is a point on the number line is either rational or irrational. Then initiate students to find a bigger number system which consist the two. That number system is a real number system. Once if you define a real number you can proceed to the sub section lessons step by step.

Start the lesson by writing different numbers which they have learned so far on the chalkboard and ask them to categorize as rational or irrational. Also check whether they can able to locate these numbers on the number line or not. Following to this give activity 1.13 to the students to imagine a number system bigger than the other two(rational and irrational).

Answer for Activity1.14

1. Yes, there should be a number system. At least they can try using diagram even without mentioning the name.
2. Since we represent both rational and irrational numbers on the number line, there is one to one correspondence between the points on the number line and the two number system (latter after giving definition with the real number system).

Answer for Exercise 1.22

a. $0.236 < 0.256$

b. $-0.1351 < -0.135$

c. $\frac{\sqrt{5}}{2} < 0.234$

d. $\pi < \frac{22}{7}$

e. $-7\sqrt{145} < \sqrt{7}$

f. $6\sqrt{5} > 5\sqrt{7}$

g. $2 + \sqrt{3} < 4$

Determining real numbers between two numbers **Period allowed: 1 period**

Answer for Exercise 1.23

1.

a. There might be different solution

Let us take the first number be $a = -0.24\ and\ b = -0.246$

Then $\frac{a+b}{2}$ is in between $a\ and\ b$. That is -0.243, you can also give other points like$-0.241, -0.244$,...

b. First let us see which number is less than to the other.

$$\frac{2}{7} < \frac{3}{5}$$

Because, by taking $\text{LCM}(5,7) = 35$, and multiply both numbers we got $35 \times \frac{2}{7} = 10\ \ and\ 35 \times \frac{3}{5} = 21$.

Then the above numbers can be written as $\frac{10}{35} < \frac{21}{35}$, so that we can select numbers with the same denominator 35 and numerator between 10 and 21. For instance, $\frac{11}{35}, \frac{16}{35}, \ldots$ are numbers between them.

c. Taking the approximate value of $\sqrt{2} \approx 1.4142 \ldots and \sqrt{3} \approx 1.7320 \ldots$
So that 1.45,1.52,1.61. be numbers between them.

2. There are infinitely many real numbers between any two real numbers

1.4.1 Intervals

Period allowed: 1 period

In this subsection you will discuss how real numbers between two points and at the right and at the left of a certain number can be expressed using intervals. To start the lesson give activity 1.14 to the students which will lead the importance of an interval to avoid listing many real numbers infinitely to the given conditions. Finally, list all interval notations using a table.

Possible teaching aids: Ruler and coloured chalk

Answer for Activity 1.15

1. (Many option) $2, 2.1, 3, 5, \ldots$
2. (Many option) $1.5, 1.6, 1.8, 2, \ldots$
3. The student may guess or some students who read other pre information may give answer in terms of interval or using number line.

Answer for Exercise 1.24

1. a. $[-3,8]$

   b. $(4,6)$

   c. $[-1, \infty)$

2. Represent each of the following intervals on the number line (draw these intervals on the number line)

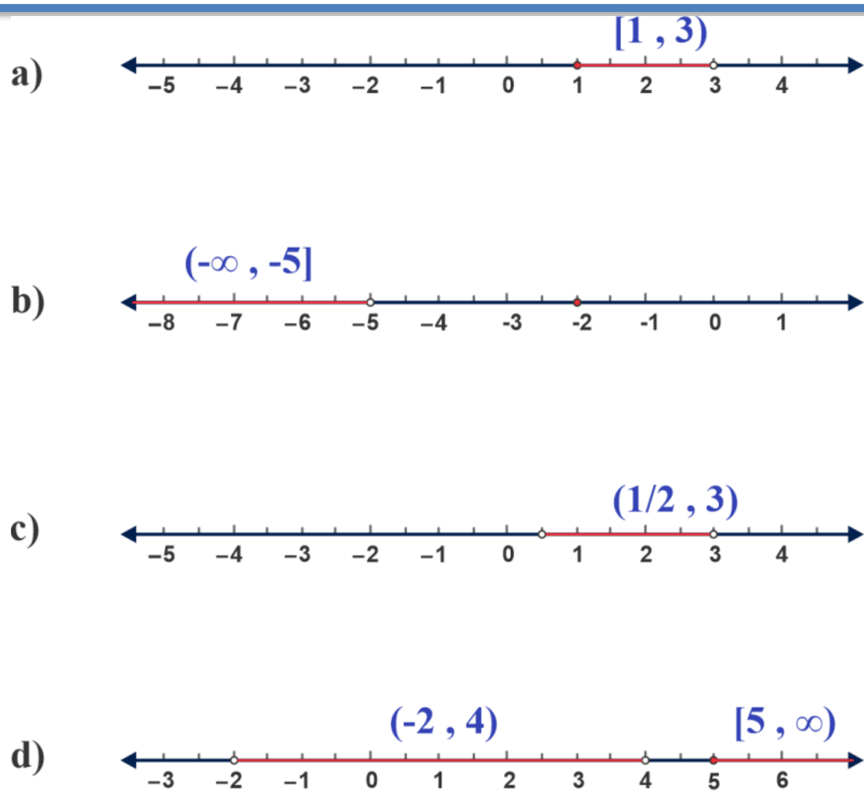


Figure 1.5

1.4.2 Absolute values

Period allowed: 1 period

The main task of this subsection is to introduce the idea of absolute value. The application of absolute value will be seen on chapter two. Hence, you can begin by giving activity 1.15 to show that the distance from the origin to the left and to the right will not have an effect on distance. After this activity deliver the formal definition of an absolute value and give appropriate examples. Finally provide Exercise 1.13.

Possible teaching aids: Ruler and coloured chalk

To start the lesson you might initiate the lesson as follows before activity 1.15

- ➢ Stand so that the table is to your right.
- **Ask students:** ‘’ How far am I from the table?’’

  Students may respond with an estimated number of meters.
- **Ask students:** If I were blind, how would you tell me where the table is?

  Students should respond with both the estimated number of meter and a direction. If students provide different units, even nonstandard ones like "steps" or "arm lengths" appreciate their participation and encourage being part of a discussion.

- Now, stand so that the table is to your left.

- **Ask students:** '' How far am I from the table now?''
  Students may respond with an estimated number of meters
- **Ask Students:** If I were blind, how would you tell me where the table is?
  Students should respond with both the estimated length and a direction.

✓ **Then you may have the following conclusion:**

When I asked how far I was from the table, you gave me a number of meters, and it didn't matter which way I was standing. But, when I asked where the table was in relation to my position, you gave me a direction as well as a number, and then it did matter which way I was standing. Today, we'll discuss absolute value. Absolute value tells you how far a number is from zero. But it doesn't tell you which way to go! It doesn't tell you what direction a number is from zero.

Answer for Exercise 1.25

1. **a.** 8 **b.** $0.\overline{12}$ **c.** $4+\sqrt{3}$ **d.** $\sqrt{2}-1$ **e.** $3-\sqrt{5}$ **f.** $7+\sqrt{5}$

2. **a.** $10-2=8$ **b.** $|-9-7|=16$ **c.** $|-100+49|=51$ **d.** $|50+50|=100$

3. **a.** $x=\pm 8$ **b.** $x=0$ **c.** There is no such x which satisfy this expression

d. $x=\pm 11$ **e.** $x=\pm 1$

4. The range of the Earth's temperature is the difference between the maximum and the minimum temperature, that is $136°\text{F}+129°\text{F}=265°\text{F}$.

1.4.3 Exponent and radicals

Allowed period:- 3 periods

Students have elementary concepts about exponents in lower grade. At this stage, you need to equip students able to explain orally or in written format a working definition of radicals using rational exponents. You will deliver lessons which lead to apply the properties of integer exponents to rational exponents developing the notation appropriate to radicals. Students should be able to rewrite expressions containing radicals in terms of exponents and reverse the concept by rewriting exponents in terms of radicals.

To start the lesson, write the multiplication of a number to itself 5 times, 6 times and so on to to large number of times to lead the importance of using exponent or power. Then define exponent or power of a real number to positive integer, then extend to integers and finally to rational numbers as described in the text. You need to ask oral question at each stage to check whether students follow the lesson properly or not.

Exponential form

Allowed period:- 1 period

Answer for Activity 1.16

1. Using the hint in the text book we have

Power form of a number	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
A number	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

2. One is the multiplicative inverse of the other, that is , $2^{-n} = \frac{1}{2^n}$
3. $a^{-n} = \frac{1}{a^n}$

Answer for Exercise 1.26

1. **a.** $5^{\frac{1}{2}}$ **b.** $7^{\frac{1}{7}}$ **c.** $3^{\frac{4}{3}}$ **d.** $\left((81)^{\frac{1}{4}}\right)^2 = 3^2$ **e.** $\left(\frac{2}{5}\right)^{\frac{1}{3}}$
2. **a.** -3 **b.** 2 **c.** $\frac{1}{5}$ **d.** 0.3

Laws of exponent

Allowed period:- 1 period

Answer for Activity 1.17

Here let us try to express each at the left hand side part a (keep b as it is to compare a and b for each case, you can also expand b and arrive to each result in corresponding a).

1. **a.** $2^3 \times 3^3 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = (2 \times 3)^3$ **b.** $(2 \times 3)^3$
2. **a.** $\frac{2^5}{2^3} = \frac{2\times2\times2\times2\times2}{2\times2\times2} = \frac{2\times2}{1} = 2^2 = 2^{(5-3)}$ **b.** $2^{(5-3)}$
3. **a.** $(3^2)^3 = 3^2 \times 3^2 \times 3^2 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} = (3^3)^2$ **b.** $(3^3)^2$
4. **a.** $(3 \times 4)^2 = \underline{3 \times 4} \times \underline{3 \times 4} = 3 \times 3 \times 4 \times 4 = 3^2 \times 4^2$ **b.** $3^2 \times 4^2$
5. **a.** $2^2 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 = 2^{(2+3)}$ **b.** $2^{(2+3)}$

Answer for Exercise 1.27

a. $5\sqrt{5}$

b. $\frac{9^{\frac{1}{3}}}{243^{\frac{1}{3}}} = \left(\frac{9}{243}\right)^{\frac{1}{3}} = \left(\frac{1}{27}\right)^{\frac{1}{3}} = \frac{1}{3}$ c. 0.2

d. $2^{\frac{3}{2}} \times 2^{\frac{1}{2}} = 2^{\left(\frac{3}{2}+\frac{1}{2}\right)} = 2^2 = 4$ e. $\frac{5\sqrt{24}\div 2\sqrt{50}}{3\sqrt{3}} = \frac{5\sqrt{24}}{2\sqrt{50}} \times \frac{1}{3\sqrt{3}} = \frac{5\times\sqrt{3}\times 2\sqrt{2}}{2\times 5\sqrt{2}} \times \frac{1}{3\sqrt{3}} = \frac{1}{3}$

f . $\sqrt[3]{0.001} + \sqrt[4]{0.0081})\div \sqrt{0.16} = \frac{0.1+0.3}{0.4} = 1$

g. $(5^{-1})^3 \times 5^{\frac{5}{4}} \times \sqrt{25} \times \left(\sqrt[3]{125}\right)^3 = \frac{1}{5^3} \times 5^{\frac{5}{4}} \times 5 \times 5^3 = 5^{\frac{9}{4}}$

h. $\left(\sqrt{0.64}\right) \times \left(\sqrt[3]{\frac{1}{64}}\right)^2 \times (32)^{\frac{1}{5}} = 0.8 \times \frac{1}{16} \times 2 = 0.1$

i. $3^{\frac{1}{4}} \times 27^{\frac{1}{4}} = (3 \times 27)^{\frac{1}{4}} = (3^4)^{\frac{1}{4}} = 1$

j . $\sqrt{5 + 2\sqrt{6}} = \sqrt{3 + 2 + 2\sqrt{6}} = \sqrt{\left(\sqrt{3} + \sqrt{2}\right)^2} = \sqrt{3} + \sqrt{2}$

$\sqrt{8 - 2\sqrt{15}} = \sqrt{5 + 3 - 2\sqrt{15}} = \sqrt{\left(\sqrt{5} - \sqrt{3}\right)^2} = \sqrt{5} - \sqrt{3}$

Therefore, $\sqrt{5 + 2\sqrt{6}} + \sqrt{8 - 2\sqrt{15}} = \sqrt{3} + \sqrt{2} + \sqrt{5} - \sqrt{3} = \sqrt{2} + \sqrt{5}$

Addition and subtraction of radicals

Allowed period:- 1 period

Answer for Activity 1.18

1. True 2. False 3. False 4. True

Answer for Exercise 1.28

a. $\left(\sqrt{12} - \sqrt{3}\right) = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

b. $2\sqrt[3]{2}$ c. 17

Operations on real numbers

Allowed period:- 1 period

Answer for Activity 1.19

Give this activity as in group and arrange a discussion among groups. Finally let each group reflect for the solution of each. Maybe students can use calculator to check that the sum of rational and irrational number is irrational. Give a stress to reach at this conclusion.

1. a. Irrational b. Irrational c. Rational d. Irrational e. Rational

   f. Irrational g. Rational h. Irrational

2. There is a lot of option

   a. $\sqrt{2}$ *and* $\sqrt{8}$

   b. Not possible (give time for student to test)

   c. Not possible

   d. $\sqrt{3}, 2$

Answer for Exercise 1.29

1. a. $x + y = 4\sqrt{2} + 7\sqrt{5} + \sqrt{2} - 3\sqrt{5} = (4+1)\sqrt{2} + (7-3)\sqrt{5} = 5\sqrt{2} + 4\sqrt{5}.$

   b. $x - y = \left(4\sqrt{2} + 7\sqrt{5}\right) - (\sqrt{2} - 3\sqrt{5}) = (4-1)\sqrt{2} + (7+3)\sqrt{5} = 3\sqrt{2} + 10\sqrt{5}.$

2. a. $2\sqrt{5} \times 4\sqrt{3} = 8\sqrt{15}$

   b. $3\sqrt{3} \times \frac{\sqrt{3}}{3} = 3$

   c. $\left(\sqrt{3} - \sqrt{2}\right) \times \left(3\sqrt{3} - 4\sqrt{2}\right) = \sqrt{3}\left(3\sqrt{3} - 4\sqrt{2}\right) - \sqrt{2}\left(3\sqrt{3} - 4\sqrt{2}\right)$

   $=9 - 4\sqrt{6} - 3\sqrt{3} + 4 \times 2$

   $=9 + 8 - 4\sqrt{6} - 3\sqrt{3}$

   $=17 - 4\sqrt{6} - 3\sqrt{3}$

3. a. $\frac{4\sqrt{6}}{2\sqrt{2}} = \frac{4}{2}.\sqrt{\frac{6}{2}} = 2\sqrt{3}$

   b. $\frac{10\sqrt{2}}{5\sqrt{18}} = \frac{10}{5}.\sqrt{\frac{2}{18}} = 2\sqrt{\frac{1}{9}} = \frac{2}{3}$

   c. $\frac{\sqrt{5}}{3\sqrt{2}\times4\sqrt{5}} = \frac{\sqrt{5}}{12\sqrt{10}} = \frac{1}{12}.\sqrt{\frac{5}{10}} = \frac{1}{12\sqrt{2}}.$

4. a. $5\sqrt{2} + 2\sqrt{3} + 3\sqrt{3} - \sqrt{2} = (5-1)\sqrt{2} + (2+3)\sqrt{3} = 4\sqrt{2} + 5\sqrt{3}$

   b. $\sqrt{145} - \sqrt{232} + \sqrt{261} = 5\sqrt{29} - 2\sqrt{2}\sqrt{29} + 3\sqrt{29}$

   $= \left(5 - 2\sqrt{2} + 2\right)\sqrt{29} = \left(7 - 2\sqrt{2}\,\right)\sqrt{29}$

Properties on real numbers **Allowed period:- 1 period**

Answer for Activity 1.20

1. **a.** $-\sqrt{3}$ **b.** π **c.** $-0.\overline{61}$ **d.** $-\frac{1}{2\sqrt{5}}$ **e.** 0 **f.** $\sqrt{2}-\sqrt{3}$

2. **a.** $-\frac{2}{\sqrt{3}}$ **b.** $\frac{1}{3.14\overline{34}}$ **c.** $\frac{1}{3\sqrt{3}-1}$ **d.** $\frac{1-\sqrt{6}}{\sqrt[3]{4}}$ **e.** $3^{-\frac{2}{3}}$

Answer for Exercise 1.30

1. Additive inverses **a.** $-\sqrt{3}$ **b.** $\frac{2}{5}$ **c.** -1.3 **d.** $-0.\overline{1}$

Multiplicative inverse **a.** $\frac{1}{\sqrt{3}}$ **b.** $-\frac{5}{2}$ **c.** $\frac{10}{13}$ **d.** $\frac{1}{0.\overline{1}}$ or 9

3. **a.** distributive property of multiplication over addition

b. associative property of addition

c. commutative property of multiplication

4 No, we can get multiplicative inverse for non-zero real number. Hence, zero has no multiplicative inverse.

5 **a.** $a*b=2a+b$ and $b*a=2b+a$, $a*b=b*a$ only for $a=b$, but it does not hold for every a and b. Hence, the given operation is not commutative.

6 $a*(b*c)=a*(2b+c)=2a+2b+c$ and

$(a*b)*c=(2a+b)*c=2(2a+b)+c=4a+2b+c$, here (*) is not associative

1.4.4 Limit of accuracy

Rounding decimal places and significant figures

Allowed period:-1 period

A measurement of different physical quantity is too much linked to our life. Students have such an experience. But measuring different quantities, in some cases we are forced to take approximate result than the exact one. Hence, in this subtopic you need to address how much the acquired result is accurate. You may formulate problems and give to students which could be submitted as a form of project related to their experience like measuring a certain field. To start the lesson, give activity 1.21 to the students and refresh their understanding about rounding.

Answer for Activity 1.21

1. **a.** 45700 **b.** 46000
2. **a.** 8.4 **b.** 8.43
3. Write the number 28.79 to three significant figures.

   Ans. 28.8

Answer for Exercise 1.31

1. Round each of the following to the nearest whole number.

a. 36 **b.** 45 **c.** 1 **d.** 2

2. Express following decimals to 1 d.p. and 2 d.p.

a. 1.9 (1 d.p.) , 1.94 (2 d.p)

b. 4.8 (1d.p) , 4.75 (2 d.p)

c. 13.0 (1 d.p) , 13.00 (2d.p)

Answer for Exercise 1.32

a. 30,000 **b.** 41,900 **c.** 4.56

Accuracy (lower and upper bound)

Allowed period:-1 period

Answer for Activity 1.22

1. 3.51,3.48,3.51,3.53, 3.45, 3.49, 3.499, 3.47 and 3.54 are rounded to 3.5,
   3.42 and 3.41 are rounded to 3.4 , 3.57 and 3.59 are rounded to 3.6.
2. 3.45 is the minimum and 3.499 is the maximum

Answer for Exercise 1.33

1 **a** lower bound 44.5 m/s (including it) and upper bound 45.5 m/s (excluding it)
(you may follow the steps)

b. $44.5 \leq v < 45.5$

2 You can follow the steps given on the text book

a. lower bound 44.5 and upper bound 45.5.

b. lower bound 12.55 and upper bound 12.65.

c. lower bound 4.225 and upper bound 4.235.

3 a. $34.65 \leq x < 34.75$ b. $21.355 \leq y < 21.365$ c. $154.1335 \leq z < 154.1345$

Effect of operation on accuracy

Allowed period:-1 period

Answer for Activity 1.23

a. $a = 5$ and $b = 2$, then the lower bound of a is 4.5 and the upper bound of a is 5.5; and the lower bound of b is 1.5 and the upper bound of b is 2.5, then

i. $4.5 + 1.5 = 6$

ii. $4.5 + 2.5 = 7$

iii. $5.5 + 1.5 = 7$

iv. $5.5 + 2.5 = 8$

The lowest value is the sum of the lower bounds and the highest value is the sum of the upper bounds

b. $a = 9$ and $b = 6$, then the lower bound of a is 8.5 and the upper bound of a is 7.5; and the lower bound of b is 5.5 and the upper bound of b is 6.5, then

i. $8.5 - 5.5 = 3$

ii. $8.5 - 6.5 = 2$

iii. $9.5 - 5.5 = 4$

iv. $9.5 - 6.5 = 3$

The lowest value is the difference of the lower bound of the first and upper bound of the second and the highest value is the difference of the upper bound of the first lower bound of the second.

Answer for Exercise 1.34

1. a. upper bound 11.7 and lower bound 11.5

b. upper bound 12.7875 and lower bound 12.0575

c. The lowest value is the difference of the lower bound of the first and upper bound of the second and the highest value is the difference of the upper bound of the first and lower bound of the second. Hence, lower bound is 8.50 and upper bound is 9.15.

d. the quotient lies between 3.4127 to 3.5574 (correct to 4d.p.) .

2. The area of the football field is determined by $A = l \times w$. The lower bound for the width is 49.45m and its upper bound is 49.55 m. Similarly the lower bound of the length of the football field is 102.55 m and its upper bound is 102.65 m.

So the lower bound for the area of the football field is 5071.0975 m^2 and its upper bound is 5086.3075 m^2 .

1.4.5 Standard notation (Scientific notation)

Allowed period:-1 period

The main objective of this sub section is to discuss about scientific notation. You are expected to show how too big or too small positive numbers could be easily expressed using significant or standard notation. You can initiate students by asking ''What is the distance between the earth and the moon in meters?'' or the question given on the students textbook.

To start the lesson, give activity 1.20, to check whether students have previous information about scientific notation and to initiate them for understanding the next lesson.

Answer for Activity 1.24

The student may give different answer for the same problem; the following could be the possible answers

i) 4.8600017×10^2 **ii)** 145.8×10^{11} **iii)** 0.06504×10^{-2} **iv)** 78.34×10^{-14}

This activity will initiate students which one is a standard way of writing a number as a multiple of 10, which will lead to have a standard notation.

Answer for Exercise 1.35

1. **a.** 1.58762×10^2 **b.** 8.9×10^{-5} **c.** 5.689700547×10^4
2. **a.** 1340000 **b.** 0.0033 **c.** 0.00000004
3.
   a. $(4.2 \times 10^3) + (1.6 \times 10^3) = (4.2 + 1.6) \times 10^3 = 5.8 \times 10^3$

   b. $(2.1 \times 10^3)(1.3 \times 10^4) = (2.1 \times 1.3) \times (10^3 \times 10^4) = 2.73 \times 10^7$

   c. $(1.5 \times 10^{-3})(3.1 \times 10^3) = (1.5 \times 3.1) \times (10^{-3} \times 10^3) = 4.65 \times 10^0$

   d. $\frac{(5.0\times10^5)}{(2\times10^{-2})} = \frac{5}{2} \times 10^{(5+2)} = 2.5 \times 10^7$

Assessment

Ask students an oral question what is a standard notation mean. And how they can able to identify whether a number is written in a standard notation or not.

1.4.6 Rationalization

Rationalization (1) Allowed period:-1 period

In this subsection you are expected to introduce how to convert real numbers with irrational denominator to a real number of rational denominator. Remind the students about the property of one, that is multiplying terms by 1 will not affect the equation. By taking different examples proceed step by step to observe the different types of integrating factors. Start the lesson by asking oral question which leads to the definition of rationalization.

Answer for Exercise 1.36

a. $\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$

b. $5\sqrt{\frac{1}{3}} = \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$

c. $\frac{2\sqrt{2}}{\sqrt{7}} = \frac{2\sqrt{2}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{14}}{7}$

Answer for activity 1.25

1. $a - b$ **2.** $a^2 - b$ **3.** $a - b^2$

Assessment

You can use Exercise 1.22 to assess the students whether they have understood the concept of rationalization or not.

Rationalization (2) Allowed period:-1 period

Answers for Exercise 1.37

a. The rationalizing factor is $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, so that

$$\frac{2}{\sqrt{3}-\sqrt{2}} = \frac{2}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{2(\sqrt{3}+\sqrt{2})}{3-2} = 2(\sqrt{3}+\sqrt{2}).$$

b. The rationalizing factor is $\frac{\sqrt{3}+1}{\sqrt{3}+1}$ so that the solution is $\frac{1+\sqrt{2}+\sqrt{3}+\sqrt{6}}{2}$.

c. The rationalizing factor is $\frac{2-\sqrt{5}}{2-\sqrt{5}}$, now multiplying both the numerator and denominator by this equation we get $\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{3(2-\sqrt{5})}{4-5} = \frac{3\sqrt{5}-6}{1} = 3\sqrt{5} - 6$.

d. Here we will do by taking the sign change on the second irrational number on the denominator. You may also follow another approach. It should do another more step if the denominator on the first trial is not rational.

$$\frac{2}{\sqrt{2}+\sqrt{3}+1} \times \frac{\sqrt{2}-\sqrt{3}+1}{\sqrt{2}-\sqrt{3}+1} = \frac{2\times(\sqrt{2}-\sqrt{3}+1)}{2-\sqrt{6}+\sqrt{2}+\sqrt{6}-3+\sqrt{3}+\sqrt{2}-\sqrt{3}+1}$$

$$= \frac{2\times(\sqrt{2}-\sqrt{3}+1)}{2\sqrt{2}}$$

$$= \frac{2\times(\sqrt{2}-\sqrt{3}+1)}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2-\sqrt{6}+\sqrt{2}}{2}.$$

1.5 Applications

Period allotted: 1 period

This subsection needs your effort to show the application of real number system. Two examples are given in the students' text book. You can add more which is relevant to their level.

Competency:

At the end of this sub unit student will be able to:

- solve mathematical problems involving real numbers

Answer for Exercise 1.38

1. Anyone can cut the string in to small pieces starting from 1cm length. But to get the maximum length, we need to find their GCF, that $\text{GCF}(78,117,351) = 39$ cm.
2. The two students start running at the same time but their speed is not the same. So after a certain time, they will meet again if both of them are keep running. This time is determined by determining their LCM.
   So $\text{LCM}(6,14) = 42$. This is to mean they meet after 42 minutes.

Summary

You have to recall the basic concepts in the unit orally. Summarize the main points of the unit. Focus on the real number system and its properties. To check whether they understood or note provide the review exercise. Arrange a tutorial class and observe their reflection on the review exercise.

Answer for Review exercise on unit one

1. a. 657, it is divisible by 3 and 9 but not divisible by 2,4,5,6,8, and 10.

b. $10{,}222$,it is divisible only by 2 .

c. $64{,}916$, it is divisible by 2 and 4 but not divisible by $3, 5, 6, 8, 9$ and 10.

2. a. 6 that is $(2+4)$ **b.** 100

3. a. 3×19 **b.** $2^3 \times 3 \times 7$ **c.** $2^3 \times 67$

4. a. $\text{GCF}(36{,}60) = 12$, $\text{LCM}(36{,}60) = 180$.

b. $\text{GCF}(84{,}224) = 28$, $\text{LCM}(84{,}224) = 672$.

c. $\text{GCF}(15{,}39{,}105) = 3$, $\text{LCM}(15{,}39{,}105) = 1{,}365$.

d. $\text{GCF}(16{,}20{,}48) = 4$, $\text{LCM}(15{,}39{,}105) = 240$.

5. $\text{GCF}(a,b) \times \text{LCM}(a,b) = ab$, answer 18.

6. a. -2.2 **b.** $1.\overline{571428}$ **c.** $0.\overline{51}$

7. a. $\frac{19}{50}$ **b.** $\frac{191}{90}$ **c.** $-\frac{41}{333}$

8. Let $x, y \in \mathbb{R}$. Suppose $x < y$. We need to show there exist $m \epsilon \mathbb{R}$ *such that* x.

$$x < y \Longrightarrow (y-x) > 0 \Longrightarrow 2(y-x) > (y-x) > 0$$

$$\Longrightarrow (y-x) > \frac{(y-x)}{2} > 0.$$

Now adding x to all sides,

$y > \frac{(x+y)}{2} > x$, here $m = \frac{x+y}{2}$ is also a real number since it is the sum of two real numbers.

So that there exist $m \epsilon \mathbb{R}$, such that $x < m < y$.

9. We need to show $\left(\frac{a}{b}\right) \div \left(\frac{c}{d}\right)$is a rational number if each of them is a rational number with the divisor is different from zero. This is to mean $c \neq 0$.

The given expression can be written as $\frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$ this is rational of rational number is closed for multiplication.

10. **a.** $10.14\overline{18}$ **b.** $2-\sqrt[3]{6}$ **c.** $-\frac{a}{b}$

11. **a.** $14^{\frac{1}{2}}$ **b.** $(x+y)^{\frac{1}{2}}$ **c.** $7^{\frac{1}{5}}$ **d.** $\left(\frac{5}{6}\right)^{\frac{1}{3}}$

12. **a.** $\frac{-2\sqrt{5}}{5}$ **b.** $\frac{4}{7}(3+\sqrt{2})$ **c.** $\sqrt{6}-2$ **d.** $\frac{1}{2}(2\sqrt{2}-\sqrt{5}-\sqrt{10}+3)$

13. $a=\frac{4}{13}, b=\frac{4}{13}, c=\frac{-1}{13}, and\ d=\frac{-3}{13}.$

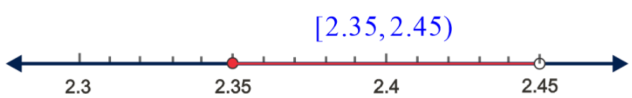
14. **a.** $6\sqrt{2}$ **b.** 1 **c.** $\frac{3}{4}$

d. $\sqrt[4]{b^2\times\sqrt[3]{b^2}}=\sqrt[4]{b^2\times b^{\frac{2}{3}}}=\sqrt[4]{b^{2+\frac{2}{3}}}=\sqrt[4]{b^{\frac{8}{3}}}=b^{\frac{8}{12}}=\sqrt[3]{b^2}$

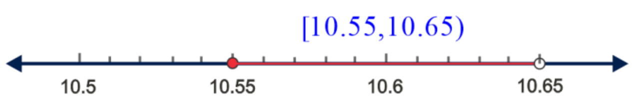
e. $\frac{\sqrt{72}-3\sqrt{24}}{\sqrt{2}}=\frac{6\sqrt{2}-6\sqrt{6}}{\sqrt{2}}=\frac{6\sqrt{2}(1-\sqrt{3})}{\sqrt{2}}=6(1-\sqrt{3}).$

f. $\frac{2\sqrt{72}}{3}-\frac{3\sqrt{128}}{4}+5\sqrt{\frac{1}{2}}=\frac{12\sqrt{2}}{3}-\frac{24\sqrt{2}}{4}+\frac{5}{\sqrt{2}}=-2\sqrt{2}+\frac{5}{\sqrt{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}.$

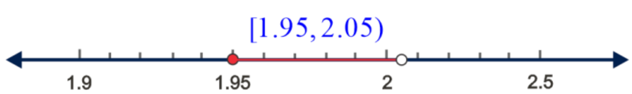
15. **a.** this number is between 2.35 and 2.45, in interval notation $2.35\le x<2.45$.



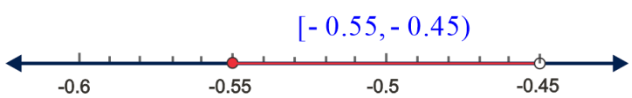
b. the number is between 10.55 and 10.65, in interval form $10.55\le x<10.65$.



c. the number is between 1.95 and 2.05, in interval form $1.95\le x<2.05$.



d. the given number is in between -0.55 and -0.45, in interval notation $-0.55<x\le-0.45$.



16. **a.** 5.672×10^8 **b.** 7.74×10^{-6} **c.** 1.546×10^7

17. The farthest real number is with largest absolute value from the origin. Also the farthest if its square is largest. Hence taking the square of each of the choice, we observed the answer is 'B'

18. Divide 736 by 12 we obtain the quotient 61 and the remainder 4 , so the answer is 'D'.

19. A is not necessarily true take for instance $x=2\ and\ y=\sqrt{3}, xy=2\sqrt{3}$ which is irrational.

B is not necessarily true, take $x = 0$ and $y = \sqrt{2}$, $\frac{x}{y} = 0$ which is rational.

C is not necessarily true since no restriction is given for x, $x$should be different from zero.

D is necessarily true

20. A is not true, since $0 \in \mathbb{R}$ which has no multiplicative inverse, B is not true and D is also not true. The answer is C

21. Dividend=divisor $\times$ quotient+remainder

$$a = 13q_1 + 9 \qquad (1)$$

$$b = 13q_2 + 7 \qquad (2)$$

$$c = 13q_3 + 10 \qquad (3)$$

Adding these three equations we get

$$a + b + c = 13q_1 + 9 + 13q_2 + 7 + 13q_3 + 10$$

$$= 13q_1 + 13q_2 + 13q_3 + 26$$

$$= 13(q_1 + q_2 + q_3) + 13(2)$$

$$\Rightarrow a + b + c = 13[q_1 + q_2 + q_3 + 2]$$

So, $a + b + c$ is divisible by 13.

22. GCF $(14,21) = 7$ groups so that in each team there are 2 girls and 3 boys.

Unit 2

Solving Equations (22 periods)

Introduction

In earlier grades, students have learned about algebraic equations and their classifications. They also have learned about linear equations in one variable and the methods to solve them. In the present unit, you discuss further about systems of linear equations in two variables, equations involving absolute values, quadratic equations in single variable, equations involving exponents and radicals and applications on equations. They shall also learn about the methods to solve them.

After completing this unit, students will be able to:

- Solve systems of simultaneous equations in two variables.
- Solve problems on equations involving exponents and radicals
- Solve simple equations involving absolute values
- Solve quadratic equations.

2.1 Revision on linear equation in one variable

Periods allotted: 1 Period

Competency

- solve a linear equation both algebraically and on a number line

Key words: Equation in one variable, linear

In earlier class, you have come across several algebraic expressions and equations. Some examples of expressions you have so far worked with were: $5x$, $2x-3$, $3x+y$, $2xy+5$, $xyz+x+y+z$, x^2+1, $y+y^2$. Some examples of equations were: $5x=25$, $2x-3=9$, $2y+\frac{5}{2}=\frac{25}{2}$ and $6z+10=-2$.

You can approach this sub-topic by letting each student to do Activity 2.1. The objective of this activity is to assist students to revise solving linear equations in one variable and recognize that a linear equation in one variable has one solution.

Answers to Activity 2.1

1. a, b, c, d, g, h, i and j are linear equations in one variable, whereas, e, f, k and l are not.
2. **a)** 2, **b)** $\frac{1}{2}$, **c)** -5, **d)** 0 **g)** $\frac{-23}{4}$, **h)** $\frac{1}{2}$ **i)** $\frac{-27}{7}$ **j)** -2

Answers to Exercises 2.1

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. **a)** $x = -7$ **b)** $x = 5$ **c)** $x = 5$ **d)** $x = -1$ **e)** $x = 2$ **f)** $x = -8$ **g)** $x = -7$
2. When we insert the value of $x = " - 3 "$ into the equation, the equation is not true. Therefore, $x = " - 3"$ is not a solution to the equation.
3. Let x *and* $x + 1$ be two consecutive numbers. Then, $x + x + 1 = 67$. So, $x = 33$. Therefore, the consecutive numbers are 33 *and* 34.

ASSESSMENT

Consider the equation $2x - 2 = 0$ and ask students to find the solution in more than one way (multiple representation of the solution, including guess, check and number line). The significance of representing the solution of linear equations in multiple ways provides the same objective of more than one form. It is necessary to see how students use these representations. It is suggested that multiple representations provide an environment for students to abstract and understand major concepts. You can assess whenever necessary.

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving linear expression, and/or tests/quizzes.

2.2 Systems of linear equations in two variables

Periods allotted: 3 Periods

Competencies

- determine and verify the solution of a system of linear equations graphically, with or without technology.

- explain the meaning of the point of intersection of a system of linear equations.
- explain, using examples, why a system of equations may have no solution, one solution, or an infinite number of solutions.
- describe a strategy to solve a system of linear equations.
- solve a contextual problem that involves a system of linear equations, with or without technology.
- relate a system of linear equations to the context of a problem.
- solve a problem by determining model a situation, using a system of linear equations.

Key words: Equations in two variables, System of equations, linear

Introduction

This subunit is dedicated to discuss systems of linear equations in two variables. Students who recalled linear equations in two variables and their solutions in section 2.1 can understand this section. Students need to recognize what happens if two or more equations are considered at a time. How to proceed to deal with this sub-unit may differ from one teacher to another but some direction that you can use as a foundation is outlined below.

Answers to Activity 2.2

Solution: Let p be the price of one pencil and e be the price of one eraser. Then we have the simultaneous equation $2p + e = 5$

$3p + 2e = 8$

Substitute $e = 5 - 2p$ into the second equation.

$$3p + 2(5 - 2p) = 8$$

$$3p + 10 - 4p = 8$$

$-p = -2$. Hence, $p = 2$. Substituting $p = 2$ into one of the original equations, you can get $e = 1$.

TEACHING NOTES

Let students to find solution to a linear equation and see different equivalent equations. Let them critically understand the difference between the solutions of a linear equation in one variable of type $3x - 1 = 8$ and linear equation in two variables of type $4x - y = 5$, as a single number and as infinitely many respectively.

At this stage students are expected to reach at a conclusion that such equations may have exactly one solution, no solution or infinitely many solutions. When the students finish the group work, let at least two groups and show their work on the board. With question and answer, assist students to arrive at the anticipated conclusion.

Give a room to the students to discuss how to find out or evaluate one of the variables given a value of the other. Assist the students to discuss and plot the points they evaluated.

Urge your students to discuss the number of solutions and notice why there are infinitely many solutions for possible plots of some points in two variables if it has a minimum of two pairs of numbers that fulfill the equation.

Your students may be required to find out the equation of a line from its plot points. You may give points, such as (0, 1), (1, 4), (2, 7) and (3, 10) and ask and expect your students whether they represent a linear line or not. This will assist students to visualize what plots of points of linear equation must look like.

Assist students to sketch two linear equations in two variables on the same $xy-plane$. Help the students to indicate the point of intersection of the equations (if any and possible) and also let students discuss if the equations do not have intersections.

Have students discuss solved problem structures and solutions to make connections among strategies, reasoning and to indicate the step(s) where there is/are a mistake(s).

Create opportunities for students to discuss and analyze solved problems by asking students to describe the steps taken in the solved problem and to explain the reasoning used. Ask students specific questions about the solution strategy, and whether that strategy is logical and mathematically correct. For example, consider $2x + 3y = 13$ which method/strategy is better than the others?

$$y = 5 - 4x$$

Because one of the equation in this system is already written as "$y =$", it makes sense to use substitution method.

Asking such questions encourages active student engagement. Vary the questions based on the needs of students and the types of problems being discussed in activities, examples, conclusions, exercises and so on. The questions can be asked verbally or written for students to refer. Consider the equation $\begin{cases} y = 2x + 3 \\ y = 2x - 4 \end{cases}$ which method/strategy is better than the others? Because the equations have equal slope and different y-intercept; they are parallel; hence, there is no solution. What about $\begin{cases} y = 2x + 3 \\ \frac{y}{2} = x + \frac{3}{2} \end{cases}$? In this case, the two equations are identical; hence, the solution is infinite.

In order to facilitate discussions, you can ask questions such as

- Can anyone think of a different strategy, (other than graphical method), to solve systems of linear equations?
- Will this strategy always work? Why?
- What are other problems for which this strategy will work?
- How can you change the given problem so that this strategy does not work?
- How can you modify the solution to make it clearer to others?

Select solved problems that reflect the lesson's instructional aim, including problems that illustrate common errors.

Use solved problems with or without common errors to accomplish diverse learning objectives

Answers to Exercises 2.2

Show for students by **checking some of the answers** using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. $y - 2x = 1$

x	0	**1**	2	3	4	5	6
y	1	**3**	5	7	9	11	13

$x + y = 4$

x	0	**1**	2	3	4	5	6
y	4	**3**	2	1	0	-1	-2

2. For the first equation we have $5(8) - 4(5) = 20$ but for the second equation $2(8) + 1 = 17 \neq 3(5) = 15$. therefore, the order pair $(8,5)$ is not a solution to the equation.

2.2.1 Solving systems of equations by substitution

Answers to Exercises 2.3

Show for students by **checking some of the answers** using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1.
   a. $x = 4,\ y = 6$
   b. $y = 6,\ x = 42$
   c. $x = 2,\ y = 15$
   d. $y = 4,\ x = -2$
   e. $x = -6, y = -2$
   f. $x = -2\ and\ y = -3$
   g. $x = 11/2\ and\ y = 7$
   h. $x = \frac{-1}{2}, y = -17$
   i. No solution
   j. Infinite solution.
   k. No solution
   l. Infinite solution
2. Let p and b be the cost of one pen and one book respectively. Then we have
   $\begin{matrix} 2p + 3b = 170 \\ 5p + b = 100 \end{matrix}$. Solving the simultaneous equation, $p = 10$ & $b = 50$

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving simultaneous equations using substitution method and/or tests/quizzes. You can assess whenever necessary.

Consider $\begin{cases} 3x + \frac{1}{2}y = 17 \\ y = 2x + 4 \end{cases}$. Ask students which equation is easier to be substituted to the other.

Ask them why? The second equation is easier to be substituted into the first because it is already written as $"y = "$

2.2.2 Solving systems of equations in two variables by Elimination method.

Answers to Exercises 2.4

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

a. $y = 1 \; and \; x = 2$

b. $x = 6 \; and \; y = 0$

c. $y = -7 \; and \; x = -8$

d. $x = -6 \; and \; y = -2$

e. No solution.

f. No solution

g. Infinite solution.

h. No solution

i. Infinite solution.

j. $x = \frac{14}{3} \; and \; y = \frac{-4}{3}$

k. $x = 6 \; and \; y = 9$

l. $x = 9 \; and \; y = -2$

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving simultaneous equations using elimination method and/or tests/quizzes.

Consider the equation$\begin{cases} 3x + y = 13 \\ -2x + 4y = 10 \end{cases}$. Ask students the numbers that each equation is multiplied so that the variable y eliminates.

Answers to Exercises 2.5

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. Let the father's age be x years and son's age be y years.

   Then $2y + x = 56$ ……………… (i)
   $2x + y = 82$…………… …(ii)
   Solving the simultaneous equation, we have $x = 36 \; and \; y = 10$

2. Let the tenth and the unit digits be $a \; and \; b \; respectively$. Then, we have

   $a + b = 13,$

   $2a = b + 1.$

Solving the equation using any one of the techniques, we have $a = 4 \; and \; b = 9$.

3. Let the tenth and the unit digits be $a \; and \; b, respectively$. Then,

$ab = 10a + b$

$3ab = 300 + 10a + b$

Multiplying by 2, we have $2(300 + 10a + b) = 27(10a + b)$, which implies

Solving for $10a + b$, we have the original number, $10a + b = 24; of couse$

$a = 2 \; and \; b = 4$

4. **a)** one solution **b)** No solution **c)** infinite solution

d) one solution **e)**one solution **f)** infinite solution

5. In order to have

i. an infinite solution, the equation should fulfill

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \; which \; implies \; that \; \frac{1}{2} = \frac{-1}{-2} = \frac{3}{k}$. Hence, $k = 6$

ii. no solution, the equation should fulfill

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \; which \; implies \; that \; \frac{1}{2} = \frac{-1}{-2} \neq \frac{3}{k}$. Hence, $k \neq 6$

iii. one solution, the equation should fulfill

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \; which \; implies \; that \; \frac{1}{2} = \frac{-1}{-2}$. Hence, there is no value of k in order for this equation to have one solution.

6. In order to have

i. an infinite solution, the equation should fulfill

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \; which \; implies \; that \; \frac{1}{2} = \frac{-1}{-2} = \frac{a}{b}$. Hence, $2a = b$

ii. no solution, the equation should fulfill

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \; which \; implies \; that \; \frac{1}{2} = \frac{-1}{-2} \neq \frac{a}{b}$. Hence, $2a \neq b$

iii. one solution, the equation should fulfill

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \; which \; implies \; that \; \frac{1}{2} = \frac{-1}{-2}$. Hence, there is no values of a and b in order for this equation to have one solution.

Note that the method we choose to solve a simultaneous equation depends on its appearance as follows:

Table 2.1

Problem statement	Solution strategy	Solution steps	Notes about strategies
$-2x + y = 7$ $x = 6y + 2$	Substitution	$-2(6y + 2) + y = 7$ $-12y - 4 + y = 7$ $-11y - 4 = 7$ $-11y = 11$ $y = -1$	Because one of the equation in this system is already written as "$x =$", it makes sense to use substitution method
$2x + y = 6$ $x - y = 9$	Elimination	$2x + y = 6$ $x - y = 9$ $3x = 15$ $x = 5, x = -4$	Because of the coefficients of the y terms are equal in absolute value but have opposite sign, the strategy of elimination may be a natural fit.

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving simultaneous equations using graphical method and/or tests/quizzes. You can assess whenever necessary.

Consider $\begin{cases} x + 3y = 12 \\ x = 2y + 2 \end{cases}$ and ask students to find the solution in more than one way (multiple representation of the solution, including guess, check and number line). Which method is easy and straight forward? Multiple representations/strategies provide an environment for students to abstract and understand major concepts.

2.3 Solving non-linear equations(15 periods)

Periods allotted: 4 Periods

2.3.1 Equations involving absolute value.

Competency

- *solve equations involving absolute values.*

Key words: Absolute value, Equations involving absolute values.

Answers to activity 2.3

a. $|2| < |-3|$ **b.** $|6| > |-5|$ **c.** $\left|\frac{1}{4}\right| < \left|-\frac{1}{2}\right|$

d. $\left|-\sqrt{10}\right| < |3|$ **e.** $\left|-\frac{1}{3}\right| = |-0.3333\ldots|$

Some measurements such as distance, perimeter and area presume positive magnitude. Attached to such conditions, discourse on absolute values is fundamental. In this sub-unit students need to discuss on absolute values first and then they need to go forward into discussing equations that involve absolute values. Properties of absolute values are also discoursed in this sub-unit.

Answers to Exercise 2.6

a. 4 **b.** $\frac{1}{2}$ **c.** 0

Answers to Exercise 2.7

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. Both of the points are 6 units from the origin.
2. Solve the following absolute value equations
   - **a.** $x = 4 \text{ and } x = -8$
   - **b.** $x = 4 \text{ and } x = \frac{-2}{3}$
   - **c.** $x = 2$
   - **d.** $x = \emptyset$
   - **e.** $x = 2 \text{ and } x = 8$
   - **f.** $x = -9 \text{ and } x = 16$
   - **g.** $x = \frac{7}{4} \text{ and } x = \frac{-9}{4}$
   - **h.** $x = 4 \text{ and } x = -10$
   - **i.** $x = 5 \text{ and } x = -23$
   - **j.** $x = -1 \text{ and } x = \frac{-3}{2}$
   - **k.** $x = 2 \text{ and } x = -3$

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving absolute values and/or tests/quizzes.

Consider the absolute value equations $|2x-6|=0$, $|4x-9|=7$ and $|3x-5|=-1$. Ask students how many solutions each of the equation has and find their solution.

2.3.2 Quadratic equation (7 Periods)

Competencies

- Derive the quadratic formula, using deductive reasoning.
- Solve a quadratic equation of the form $ax^2+bx+c=0$ by using strategies such as determining square roots, factorizing, completing the square and applying the quadratic formula
- Identify relationship between roots and coefficients of a quadratic equation
- Solve a problem by determining or analyzing a quadratic equation.

Answers to activity 2.4

1. $x^2+x-6=0$ **2.** $10x^2+22x-2=0$ **3.** $\frac{1}{2}x^2-\frac{1}{2}x-15=0$ **4.** $-x^2+\frac{17}{2}x-4=0$

Here are some possible directions or helpful suggestions for helping students in solving quadratic equations.

Factoring is often done by guessing and checking, which can be time consuming, depending on the problem. **Graphing** (for example fig 2.4) can be done by hand or by using technology, and the choice might depend on such things as whether the intercepts are known to be integers or irrational numbers.

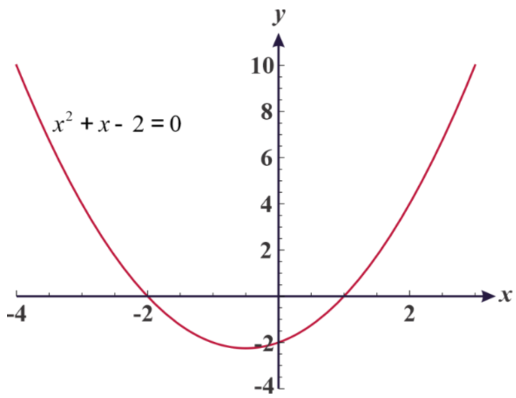


Figure 2.4

Tables may be useful in illustrating how changes in one variable are related to changes in the other variable. Start by substituting $x = -3$. Then $x = -2$ for which it allows the value equal to zero. Knowing that the value of $x^2 + x - 2$ decreases then increases as the value of x increases which suggests that we should continue by substituting -1 for . Symmetry in the values becomes apparent after substituting 1 for , leading to the discovery of the second zero.

Table 2.2

$x^2 + x - 2 = 0$	
x	$x^2 + x - 2$
-3	4
-2	0
-1	-2
0	-4
1	0
2	4
3	11

However, completing the square method always works, though it might be difficult to handle. Although completing the square may be a relatively complex algebraic process, this method is very useful in helping students notice the mathematical structure that unites quadratic equations with squared quantities.

Answers to activity 2.5

a. 2 & 3 **b.** 4 & -3

Answers to Exercise 2.8

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

a. $x = 0 \; or \; x = 5$ **b.** $x = -2 \; or \; x = -5$ **c.** $x = -3 \; or \; x = 2$ **d.** $x = 3 \; or \; x = 1$

Answers to Exercise 2.9

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

a. $x = -5$ **b.** $x = 4$ **c.** $x = 2 \text{ or } x = -2$ **d.** $x = \frac{1}{3}$

Answers to Exercise 2.10

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

a. $x = -1$ **b.** $x = -2 \pm \sqrt{3}$ **c.** $x = 3 \pm \sqrt{14}$

d. $x = \pm\sqrt{6}$ **e.** $x = -1 \text{ or } x = -\frac{3}{2}$ **f.** No solution

Answers to Exercise 2.11

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

a. $x = \frac{-3\pm\sqrt{5}}{2}$ **b.** $x = \frac{-5\pm\sqrt{33}}{2}$ **c.** $x = \frac{3\pm\sqrt{17}}{4}$

Answers to Exercise 2.12

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

a. $No\ real\ root$ **b.** $x = -6$ **c.** $x = -7 \text{ or } x = -1$

Answers to Exercise 2.13

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. $b = -10 \quad c = 16$

2. $x^2 + 3x + 2 = 0$

a. $x = 4 \text{ or } x = -4$ **b.** $x = 0 \text{ or } x = 9$

c. $x = 5 \text{ or } x = -5$ **d.** $x = 1 \text{ or } x = 5$

e. $-2 \text{ or } x = 4$ **f.** $x = 4 \text{ or } x = -7$

g. $x = -3 \text{ or } x = -\frac{1}{2}$ **h.** $x = -3 \text{ or } x = 5$

i. $= -5 \pm 3\sqrt{2}$ **j.** $x = 1 \text{ or } x = \frac{7}{3}$

k. $x = 3 \pm \sqrt{6}$ **l.** No real root

m. $x = 1 \text{ or } x = \frac{5}{3}$

n. $x = -4 \text{ or } x = \frac{3}{2}$

o. $x = \frac{3}{4} \text{ or } x = \frac{1}{2}$

p. $x = -\frac{1}{2} \text{ or } -2$

q. No real root

r. No real root

s. $x = 2 \text{ or } x = -\frac{7}{3}$

t. $x = \frac{9 \pm \sqrt{233}}{18}$

u. $x = \frac{3 \pm \sqrt{33}}{2}$

w. $x = \frac{-2 \pm \sqrt{508}}{14}$

x. $x = \frac{-7 \pm \sqrt{21}}{2}$

Answers to Activity 2.6

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

Answers to Exercise 2.14

Motivate students to check the answers using Geogebra, Mathlab, mathematica or similar tools. Appreciate students to use IT in mathematics.

1. a) $x = -3 \text{ and } x = -5$ b) $x = -3$ c) No real root

   d) $x = -1 \text{ and } x = \frac{-3}{2}$ e) $x = -1 \text{ and } x = \frac{4}{3}$

2. Find the values of k for which the quadratic expression $(x-k)(x-10)+1$ has integral roots.

   The given equation can be rewritten as $x^2 - (k+10)x + 10k + 1$.

   D = $b^2 - 4ac$ = $k^2 - 2k + 96 = (k-10)^2 - 4$ .The quadratic equation will have integral roots, if the value of discriminant > 0, D is a perfect square, a = 1 and b and c are integers.

   i.e. $(k-10)^2 - D = 4$. Since discriminant is a perfect square. Hence, the difference of two perfect square in R.H.S will be 4 only when D = 0 and $(k-10)^2 - D = 4$.

   Therefore, $k - 10 = \pm 2$. Therefore, the values are $k = 8 \text{ and } k = 12$.

3. Find the values of k such that the equation $\frac{p}{(x+r)} + \frac{q}{(x-r)} = \frac{k}{2x}$ has two equal roots. The given quadratic equation can be rewritten as:

   $(2\text{p} + 2\text{q} - \text{k})^2 - 2r(p-q)x + r^2k = 0$

For equal roots, the discriminant D = 0, i.e. $b^2 - 4ac = 0$

Here, $a = 2p + 2q - k$, $b = -2r(p-q)$ and $c = r^2k$

Since $r \neq 0$, therefore, after simplification one can reach $k^2 - 2(p+q)k + (p-q)^2 = 0$.

This is a quadratic equation in k. Solving for k in terms of p and q using the quadratic formula, we have

$k = \frac{2(p+q) \pm \sqrt{(p+q)^2 - 4(1)(p-q)^2}}{2}$, you can further simplify.

4. Find the quadratic equation with rational coefficients when one root is $\frac{1}{2+\sqrt{5}}$.

If the coefficients are rational, then the irrational roots occur in conjugate pairs.

Therefore, if one root is $\alpha = \frac{1}{2+\sqrt{5}} = \sqrt{5} - 2$, then the other root will be

$\beta = \frac{-1}{-2+\sqrt{5}} = -\sqrt{5} - 2$. Sum of the roots $\alpha + \beta = -4$ and product of roots $\alpha\,\beta = -1$. Thus, the required equation is $x^2 + 4x - 1 = 0$

5. If the coefficient of x is 17, then with roots -2 and -15 , the equation will be

$x^2 - (x+2)(x+15) = 0$, implies $x^2 + 17x + 30 = 0$. Therefore, $c = 30$.

$Now, let\ us\ change\ the\ coefficient\ of x = 17\ by\ 13$.

Then the roots of the original quadratic equations equation is $x^2 + 13x + 30 = 0$.

Solving this quadratic equation, we get $x = -3$ and $x = -10$. Therefore; the roots of the original quadratic equations are -3 and -10.

6. The roots of $6x^2 - 17x + 12 = 0$ are $r_1 = \frac{3}{2}$ and $r_2 = \frac{4}{3}$. If one of the roots say $r_1 = 3/2$ is also one of the roots of $3x^2 - 2x + k = 0$, then using sum of the roots, the other root is $r_2 = \frac{-5}{6}$. Hence, using product of the roots, $k = \frac{-15}{4}$. The other round can be done similarly.

7. Find the values of k such that the quadratic equations $x^2 - 11x + k = 0$ and $x^2 - 14x + 2k = 0$ have a common factor.

Let $(x - \alpha)$ be the common factor of the quadratic equations $x^2 - 11x + k = 0$ and $x^2 - 14x + 2k = 0$. Then, $x = \alpha$ will satisfy the given quadratic equations.

Therefore, $\alpha^2 - 11\alpha + k = 0$ . . . . (1)

And, $\alpha^2 - 14\alpha + 2k = 0$ . . . . (2)

On Solving Equation (1) and Equation (2), we will get:

Therefore, $\alpha^2 = \frac{8k}{3}$ . . . .(3) and, $\alpha = \frac{k}{3}$ . . . . (4)

On equating equation (3) and equation (4):$(\frac{k}{3})^2 = \frac{8k}{3}$. Therefore, the value of $k = 24$.

8. When we add and subtract the two equations, we get the following:

$x^2 + y^2 = 18x + 18y = 18(x + y)$

$x^2 - y^2 = (x - y)(x + y) = 16x - 16y = 16(x - y)$

$x + y = 16$ (since$x \neq y$)

Therefore, $x^2 + y^2 = 18(16) = (17 + 1)(17 - 1) = 17^2 - 1^2 = 17^2 - 1$

Hence, $x^2 + y^2 + 1 = 17^2 - 1 + 1 = 17^2$. This implies that $\sqrt{x^2 + y^2 + 1} = 17$

9. $|x^2 + 2x - 4| = 4$. Remember properties of absolute value, that

$Case$ 1: $x^2 + 2x - 4 = 4$, solving the quadratic equation, we get $x = 2, -4$

$Case$ 2: $x^2 + 2x - 4 = -4$, solving the quadratic equation, we get $x = 0, -2$.

Therefore, the solution of the original equation is $x = 2, -4, 0, -2$

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving quadratic equations and/or tests/quizzes.

Consider the quadratic equations $(x + 3)(2x - 1) = 0$, $x^2 + 4x - 3 = 0$, $2x^2 + x - 1 = 0$ and $-4x^2 + 3x - 5 = 0$

Ask students to solve each by any method they choose and ask them why they choose that strategy /method?

2.3.3 Equations involving exponents and radicals

Periods allotted: 4 Periods

Competency

- *solve equations that involve exponents and radicals by applying law of exponents.*

Key words: Equations, Power, Exponents, Radicals, Bases

Answers to Activity 2.7

Equation **a)** has two solutions: $x = 2$ and $x = -2$ since $2^2 = 4$ and $(-2)^2 = 4$.

Equation **b)** has only one solution: $x = 3$ since $3^3 = 27$. Note that $x = -3$ cannot be a solution. Equation **c)** has only one solution: $x = -4$ since $(-4)^3 = -64$. Note also that $x = 4$ cannot be a solution. **d)** has only one solution: $x = 2$. Whenever an equation contains all even exponents, you should consider both the positive and negative solutions. If the exponent is an odd power, there is only one solution.

Note: Let $x^n = k$. If n is even, $x = \pm\sqrt[n]{k}$. If n is odd, $x = \sqrt[n]{k}$.

In general, $x^{n/m} = (\sqrt[m]{x})^n$ and, If $B^M = B^N$, *then* $M = N$.

Teaching Notes

Students are expected to have some background on exponents and radicals. You may ask students to present and describe some of the rules of exponents they studied in unit one. After deliberation by students, you may start this lesson with introducing one of the rules for exponents that states "for $a > 0, a^x = a^y$, if and only if $x = y$". It is possible to encourage students, through question and answer, to revise the terms such as **power**, **base** and **exponent**, and illustrate with examples from real numbers which they have learned in earlier grades.

Distribute Solving Radical Equations (Introductory Exercise) activity sheet. Have students complete the activity, working individually and then in pairs to share and confirm or revise their responses. Emphasize on the justification of their answers. Follow with a class discussion of each problem. Pay attention to false statement that leads to wrong conclusion.

To explore the algebraic and graphical methods for solving rational expressions, begin with the algebraic. Distribute the Steps for Solving Radical Equations Algebraically. As you lead students through the examples, encourage students to work with their partners to monitor and communicate what is happening. After each example, have a student pair come up and work the similar, accompanying problem. The variety of problems is meant to encompass the scope of typical problems.

Use some of the problems from the Steps for Solving Radical Equations Algebraically to introduce students to solving radical equations by graphing. Have students graph $f(x) = \sqrt{3x - 5}$. Ask students about the domain of the function. Have the students evaluate (3). Ask students to find the value of x when $(x) = 2$. Discuss how these two are related on the graph.

Ask the students to find the solution to the equation $2 = \sqrt{3x-5}$ using the graph and to describe how they could use the graph to solve the equation. Have students use a graph to find the solution to $\sqrt{x+2} = x$. Discuss how this problem is similar to and different from the first example. Remind the students of the importance of identifying the domain for each function and how that relates to the possible solutions to the equation and any extraneous solutions. When you begin to explain equations with exponents and radicals it will be fundamental to revisit some of the laws of exponents discussed in the previous grades. It is also recommended to look for practical applications whereby students can easily capture the meaning and get deeper understanding.

Note: questions like Example 9, 10 and 11, there are some values of x which cannot be a solution. This time, we need to observe our domain and check the answers.

Answers to Exercise 2.15

a. $x = 4$ **b.** $x = -3$

Answers to Exercise 2.16

a. $x = \frac{5}{2}$ **b.** $x = 3$ **c.** $x = 1$

Answers to Exercise 2.17

a. $x = \frac{-2\pm\sqrt{68}}{2}$ **b.** $x = 1$

Answers to Exercise 2.18

a. $x = 1$ **b.** $x = 2$ **c.** $x = -4$ **d.** $x = 4$

e. $x = 32$ **f.** $x = 9$ **g.** $x = -16$ **h.** $x = \frac{13}{4}$

i. $x = -1$ **j.** $x = \frac{17}{10}$ **k.** $x = 1 \text{ or } x = 2$ **l.** $No\ solution$

m. Squaring both sides twice, and then solving the quadratic equation, we have $x = -2$ as a solution.

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving exponents and radicals and/or tests/quizzes.

Consider the following equations $x^2 = 9, 5^x = 125$, $x^3 = -81, x^2 = -16$. Ask students that which of the quetions have one solution, two solution and no solution. why? Ask them to find the solution.

2.4 Applications of equations

Periods allotted: 3 Periods

Competency

- *solve equations involving real-life applications.*

Key words: Equations, applications.

Answers to Activity 2.8

Let $x =$ number of children and $y =$the number of adults who had a ticket for a wedding cermony.

The total number of people is 1,650. We can use this to write an equation for the number of people in the meeting.

$$x + y = 1{,}650$$

The money collected from all children can be found by multiplying Birr 4.00 by the number of children,$4x$. The money collected from all adults can be found by multiplying Birr 12.00 by the number of adults,$12y$. The total revenue is Birr 70,000. We can use this to write an equation for the revenue.

$$4x + 12y = 14{,}200$$

We now have a system of linear equations in two variables.

$x + y = 1{,}650$

$4x + 12y = 14{,}200$

In the first equation, the coefficient of both variables is 1. We can quickly solve the first equation for either y or x. Let us solve for y.

$x + y = 1{,}650$

$y = 1{,}650 - x$

Substitute the expression $1{,}650 - x$ in the second equation for a and solve for c .

$4x + 12(1{,}650 - x) = 14{,}200$

$4x + 19{,}800 - 12x = 14{,}200$

$-8x = -5600. Hence, x = 700$

Substitute x= 700 into the first equation to solve for y.

$700 + y = 1{,}650. Therefore, y = 950$

We find that 700 children and 950 adults bought meal tickets in the meeting that day.

Answers to Exercise 2.20

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. Solution:

Let $c =$ the number of children and $a =$ the number of adults in attendance.

The total number of people is 25,000. We can use this to write an equation for the number of people at the circus that day.

$c + a = 25{,}000$

The money collected from all children can be found by multiplying Birr 50.00 by the number of children, $50c$. The money collected from all adults can be found by multiplying Birr 75.00 by the number of adults, $75a$. The total revenue is Birr 1,375,000. We can use this to write an equation for the revenue.

$50c + 75a = 1{,}375{,}000$

We now have a system of linear equations in two variables.

$$c + a = 25{,}000$$
$$50c + 75a = 1{,}375{,}000$$

In the first equation, the coefficient of both variables is 1. We can quickly solve the first equation for either c and a. Let us solve for a.

$a = 25{,}000 - c$

Substitute the expression $2{,}000 - c$ in the second equation for a and solve for c.

$$50c + 75(25{,}000 - c) = 1{,}375{,}000$$
$$-25c = -500{,}000$$
$$c = 20{,}000$$

Substitute $c = 1200$ into the first equation to solve for a.

$20{,}000 + a = 25{,}000$

$a = 5{,}000$

2. Abdullah is choosing between two car-rental companies. The first, "Keep on carefully driving", charges an up-front fee of Birr 15, then 61 cents a kilometer. The second, "Atiften Tidesaleh", charges an up-front fee of Birr 12, then 65 cents a kilometer. When will "Keep on carefully driving" be better choice for Abdullah?

Solution:

The two important quantities in this problem are the cost and the number of kms driven. Because we have two companies to consider, we will define two functions.

Input	d, distance driven in kms
Outputs	$K(d)$: cost, in Birr, for renting from "Keep on carefully driving" A(d) cost, in Birr, for renting from "Atiften tidersaleh"
Initial Value	Up-front fee: $K(0) = 15$ and $A(0) = 12$
Rate of Change	$K(d)$ = Birr 0.61/kms and $P(d)$ = Birr 0.65/kms

A linear function is of the form $f(x) = mx + b$. Using the rates of change and initial charges, we can write the equations. Avoid the graph for the red & Sketch the graph for the purple

$K(d) = 0.59d + 20$ $\qquad$ $K(d) = 0.61d + 15$

$A(d) = 0.63d + 16$ $\qquad$ $A(d) = 0.65d + 12$

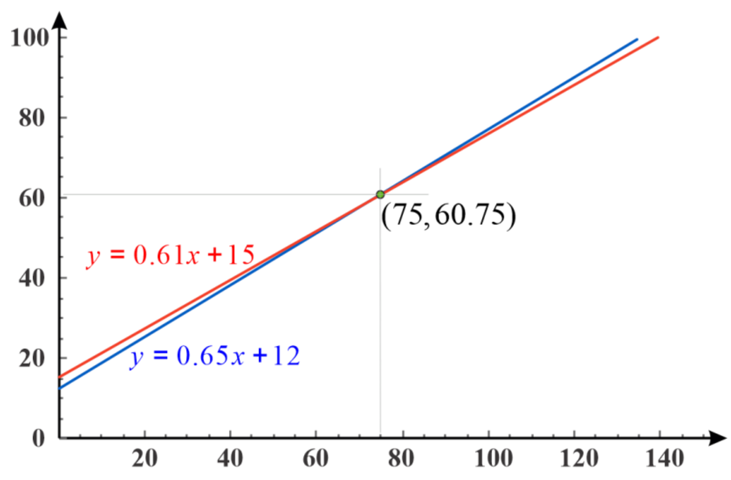


Figure 2.5

Using these equations, we can determine when "Keep on carefully driving" will be better choice. Because all we have to make that decision is the costs, we are looking for when "Atiften tidersaleh" will cost less, or when $K(d) < A(d)$. The solution pathway will lead us to find the equations for the two functions, find the intersection, and then see where the $K(d)$ function is smaller.

This tells us that the cost from the two companies will be the same if 75 kms are driven. Either by looking at the graph, or noting that $K(d)$is growing at a slower rate, we can conclude that "Keep on carefully driving" will be the cheaper price when more than 75 kms are driven, that is $d > 100$ (fig 2.5)

3. The cost of two tables and three chairs is Birr 705. If the table costs Birr 40 more than the chair, find the cost of the table and the chair.

Solution:

The table cost Birr 40 more than the chair.

Let us assume the cost of the chair to be x.

Then the cost of the table $= Birr(40 + x)$

The cost of 3 chairs $= 3(x) = 3x$ and the cost of 2 tables $= Birr\ 2(40 + x)$

Total cost of 2 tables and 3 chairs $= Birr\ 705$

Therefore, $3x + 2(40 + x) = 705$

Hence, $x = 125\ and\ 40 + x = 40 + 125 = 165$

$80 + 2x + 3x = 705$

Therefore, the cost of each chair is Birr 125 and that of each table is Birr 165.

4. A bank loaned out Birr 29,500, part of it at the rate of 6% annual interest, and the rest at 13% annual interest. The total interest earned for both loans was Birr 2,820.00. How much was loaned at each rate?

Solution:

Let $L_1\ \ and\ \ L_2$ be loans for 6% and 13% respectively. Solve the following system of equation

$$.06L_1 + 0.13L_2 = 2{,}800$$

$$L_1 + L_2 = 29{,}500$$

Solving the equation simultaneous, we have $L_1 = 14{,}500\ \ \&\ \ 15{,}000$

5. Two persons A and B together can do a piece of work in 8 days. A alone does the same work in 12 days. Then if B alone works, in how many days he can do the same work?

Solution:

If A works alone, he can complete $\left(\frac{1}{12}\right)^{th}$ of the whole work in one day. Similarly, If A and B work together, they can complete $\left(\frac{1}{8}\right)^{th}$ of the whole work in one day. Hence, if B works alone, he can complete $\left(\frac{1}{8}\right)^{th} - \left(\frac{1}{12}\right)^{th} = \left(\frac{1}{24}\right)^{th}$ of the whole work in one day. Therefore, B can complete the whole work in 24 days.

6. Three people A, B, and C can do a piece of work in 20, 30 and 60 days respectively. In how many days can A complete the work if he is assisted by B and C on every third day?

Solution:

If A, B and C each works alone, they can complete $\left(\frac{1}{20}\right)^{th}$, $\left(\frac{1}{30}\right)^{th}$ and $\left(\frac{1}{60}\right)^{th}$ of the whole work in one day respectively. Every 3^{rd} day, all will be working together (B and C help A). First and second day, A works alone, hence, he completes only $\left(\frac{1}{20}\right)^{th}$ of the whole work in one day. Therefore A completes, $\left(\frac{1}{20}\right)^{th} + \left(\frac{1}{20}\right)^{th} =$ $\left(\frac{1}{10}\right)^{th}$ $of\ the\ whole\ work\ \ in\ the\ first\ two\ days.$ In the third, sixth, ninth….days all work together, and hence complete $\left(\frac{1}{20} + \frac{1}{30} + \frac{1}{60}\right)^{th} =$ $\left(\frac{1}{10}\right)^{th}$ $of\ the\ whole\ work\ in\ one\ day.$ Therefore, A completes the whole work in 15 days.

7. A ball is shot into the air from the edge of a building 50 m above the ground. Its initial velocity is 20 meters per second. The equation $h(t) = -16t^2 + 20t + 50$ can be used to model the height of the ball after t seconds. About how long does it take for the ball to hit the ground?

Solution:

In order to hit the ground, h has to be zero; therefore, we need to solve the quadratic equation $-16t^2 + 20t + 50 = 0.$ After solving this quadratic equation, we have $t = 2.5$

It means that the ball hits the ground after 2.5 seconds.

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving application of equations and/or tests/quizzes.

Consider the following question: The cost of three books and twenty pens is Birr 500. If the pen costs Birr 90 less than the cost of the book, find the cost of one book and the cost of one pen.

$Let\ B\ be\ the\ cost\ of\ one\ book\ and\ P\ be\ the\ cost\ of\ one\ pencil.$ Then we have the following system of linear equation in two variables.

$$\begin{cases} 3B + 20P = 500 \\ P = B - 90 \end{cases}$$

Solving simultaneously, we have the cost of one book Birr100 and the cost of one pen is Birr 10

Answers to Review Exercises on Unit 2

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. T **2.** F **3.** T **4.** T **5.** F **6.** a **7.** b **8.** d

9. $x = -1$ **10.** $x = 1$ **11.** $x = \frac{-1}{3}$ **12.** $x = \frac{1}{3}$ **13.** $x = 2$ **14.** $a = 16$

15. $x = 7y - 11$

$5(7y - 11) + 2y = -18$

$35y - 55 + 2y = -18$

$37y = 37$

$y = 1\ implies\ x = 7y - 11 = 7(1) - 11 = -4$

16. Consider the second equation, $2x - y = -12$ implies $y = 2x + 12$

$-4x + 2(2x + 12)= 3$ (Substituting into the first equation)

$\Rightarrow 24 = 3$, which is not true. Hence, there is no solution.

$7x - 16x = 0\ implies - 9x = 0. We\ get\ x = 0, then\ y = \frac{3}{2}$

17. $9y = -6 - 3x\ implies\ x = 2 - 3y$

$-4x - 12y = 8$ $implies$ $x = -3y - 2$. We get $2 - 3y = -3y - 2$

$Hence, there\ is\ no\ solution$

18. $\begin{cases} 6x - 5y = 8 \\ -12x + 2y = 0 \end{cases}$

$\begin{cases} 12x - 10y = 16 \\ -12x + 2y = 0 \end{cases}$ $implies - 8y = 16. Then\ we\ get\ y = -2,\ x = \frac{1}{3}$

19. $\begin{cases} -x + 5y = 2 \\ 5x - 25y = -10 \end{cases}$ If you multiply the first equation by -5, then you will get the second equation. So, we have infinite solution.

20. $\begin{cases} 2x + 3y = 20 \\ x + \frac{3}{2}y = 10 \end{cases}$ If you multiply the first equation by $\frac{1}{2}$, then you will get the second equation. So, we have infinite solution.

21. Let the two digits be $x\ and\ y$.

$x + y = 13.$

$xy = 42.$

$From\ the\ first\ equation\ we\ get\ y = 13 - x$. Then substituting this into the second equation, we have $x(13 - x) = 42$.

$x^2 - 13x + 42 = 0$. Then the solution is $x = 6\ and\ x = 7$

22. Let G and C be the ages of Gaddisa and Chala respectively.

$G = 2C + 4$

$C = \frac{1}{3}G + 2$

Using substitution, $G = 2\left[\frac{1}{3}G + 2\right] + 4\ implies\ G = 24\ and\ C = 10$

So, the present ages of Gaddisa and Chala are $G = 24\ and\ C = 10\ respectively$.

23. $x = 3\ and\ x = \frac{1}{2}$

24. $x = 2\ and\ x = \frac{-1}{2}$

25. $x = 0\ and\ x = 1$

26. $x = \frac{21}{2}$

27. $x = 2\ and\ x = -2$

28. No solution

29. $x = 129$

30. $x = -1$

31. $x = -8$

32. a. $x = 0, x = 3$ **b.** $x = 0, x = -9$ **c.** $x = 2, x = \frac{1}{2}$ **d.** $x = \frac{1}{2}, x = -\frac{1}{3}$

e. $x = 1 \; and \; x = \frac{1}{5}$ **f.** No solution **g.** $\frac{7 \pm \sqrt{57}}{8}$ **h.** No solution

33. $\left(\frac{1}{7}\right)^{th}$ $of\ the\ whole\ pool\ will\ be\ filled\ in\ 1\ hr\ by\ the\ first\ pump.$

$\left(\frac{1}{12}\right)^{th}$ $of\ the\ whole\ pool\ will\ be\ filled\ in\ 1\ hr\ by\ the\ second\ \ pump.$

$Therefore, \left(\frac{1}{7} + \frac{1}{12}\right)^{th}$ of the whole pool will be filled in 1 hr by both the first and the second pool together. Hence, the pool will be filled by both the pumps together in $\frac{84}{19}$ seconds

34. Let w be the width of the fence. Hence, the length of the fence will be 3w(the length of rectangle). Therefore, $w + 3w = 100\ implies\ w = 25\ and\ the\ length\ l = 3w = 75$

35. Let c and m represent the cost of a single coffee and makiato respectively. Then we have

$$\begin{cases} 3c + 2m = 17 \\ 2c + 4m = 18 \end{cases}$$

Solving the simultaneous equation, we get $c = 4\ and\ m = 2.5$. Hence, the individual price for a single coffee and a single makiato are Birr 4 and 2.5 respectively.

36. C **37.** B **38.** B

Unit 3

SOLVING INEQUALITIES (20 periods)

Introduction

Inequalities are mathematical expressions involving the symbols $>, <, \geq$ *and* $\leq$. To 'solve' an inequality means to find a range, or ranges, of values that unknown x can take and moreover fulfill the inequality. In this unit, inequalities are solved by using different techniques such as algebraic operations and drawing graphs. In order to master the techniques explained here, it is critical that you undertake plenty of practices on exercises and problems on applications on inequalities.

After completing this unit, students will be able to:

- solve linear inequalities in one variable.
- solve system of linear inequalities in two variables.
- solve inequalities involving absolute value.
- solve quadratic inequalities.
- apply inequalities in real life situations.

3.1 Revision on Linear Inequalities in one Variable

Periods allotted 2

Competency

- *solve linear inequalities in one variables.*

Key words: Inequalities, Linear, Revision

Ask students to draw and describe the graphs of linear inequalities in one variable. Let them perform **Activity 3.1**. Emphasize that one of the half-planes contain the solutions of the linear inequality. Use solid line if the symbol ≥ or ≤ is used and broken line if the symbol used is > or < is used.

Answers to Activity 3.1

1.a No change

b If you multiply by a negative number, the inequality signs change. If you multiply by a positive number the sign does not change.

If math software like GeoGebra is available, ask the students to make use of this. GeoGebra is dynamic mathematics software that can be used to visualize and understand concepts in algebra, geometry, calculus, and statistics.

Answers to Exercise 3.1

a. $x > 4$ **b.** $x < 3$ **c.** $x \geq -10$ **d.** $x \geq 2$

Answers to Exercise 3.2

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. $x \geq 2$
2. $x \geq 11$
3. $x < 8$
4. $x < \frac{11}{5}$
5. $x \leq -16$
6. $x < 22$
7. $y \leq 9$
8. $m > \frac{6}{7}$
9. $m > \frac{-19}{3}$
10. $x > \frac{-11}{4}$
11. $x > -\frac{3}{2}$
12. $x > 1$
13. $x \geq -1$
14. $x \geq 3$
15. $x \geq -\frac{1}{4}$

ASSESSMENT

Consider the inequalities$-4 \geq 0$, $3x + 5 \leq 4$, $-x - 6 \leq 1$and $-\frac{1}{2}x + 5 > 4$. Ask students to find the solution in more than one way including number line, as assessment for learning. Moreover, ask students how many solutions each inequality has. You can assess whenever necessary.

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving linear expression, and/or tests/quizzes.

3.2. Systems of Linear Inequalities in two Variables

Periods allotted 4

Competencies

- solve system of linear inequalities in two variables by using algebraic operations.
- solve system of linear inequalities in two variables by using graphical method.

Key words: Inequalities, System, Linear, Two variables.

Let the students determine the linear inequality whose graph is described by a shaded region. Ask them to perform **Activity 3.2.** Encourage them to use different ways of finding the linear inequality. In this activity, one possible error that students might commit is the wrong use of inequality symbol. Let them check their own errors by testing some ordered pairs against the inequality they have formulated. Emphasize to them also the meanings of the broken and solid lines.

Answers to Activity 3.2

1. $Using\ the\ general\ form\ of\ linear\ equation\ y = mx + b$, where m is the slope and b is the y-intercept. Moreover, the x-intercept is $\frac{-b}{m}$

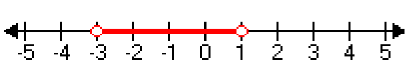
2. On the left side of $y = x + 3$, $y > x + 3$ & on the right side of $y = x + 3$, $y < x + 3$ Those on the line $y = x + 3$ is $y = x + 3$

3. c. $-x + y \geq 2$

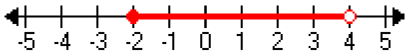
If math software like GeoGebra is available, ask the students to make use of this. GeoGebra is a dynamic mathematics software that can be used to visualize and understand concepts in algebra, geometry, calculus, and statistics.

Answers to Exercise 3.3

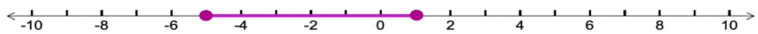
1. **a.** $[-3, 2]$ **b.** $[2, 7)$ **c.** $(-5, 0]$



2. **a)**

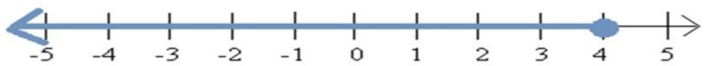


b)



c)

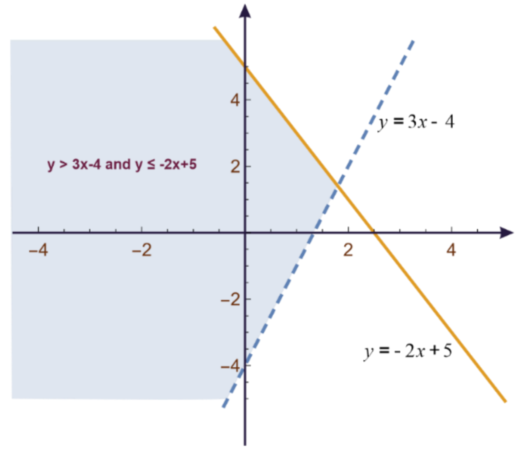
3.



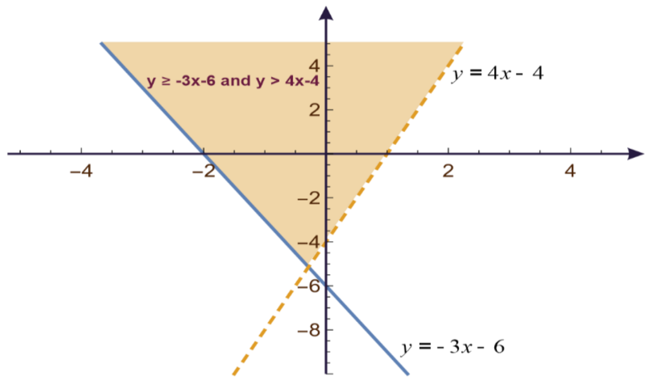
Answers to Exercise 3.6

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

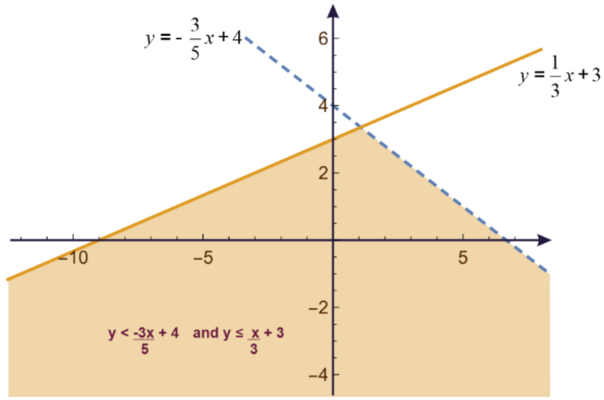
1. $\begin{cases} y > 3x - 4 \\ y \leq -2x + 5 \end{cases}$



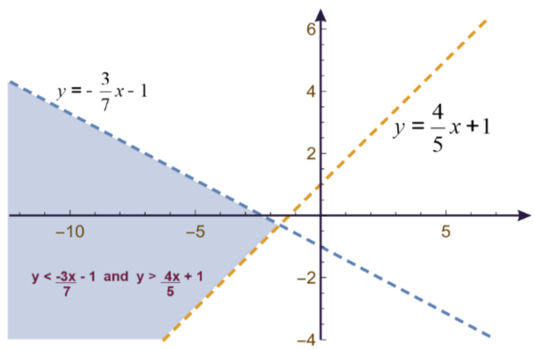
2. $\begin{cases} y \geq -3x - 6 \\ y > 4x - 4 \end{cases}$



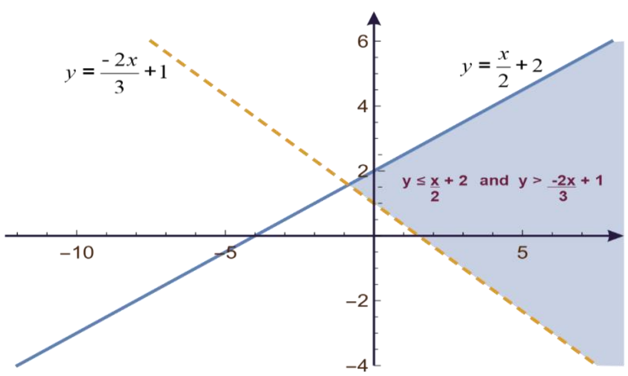
3. $\begin{cases} y < \dfrac{-3}{5}x + 4 \\ y \leq \dfrac{1}{3}x + 3 \end{cases}$



4. $\begin{cases} y < \frac{-3}{7}x - 1 \\ y > \frac{4}{5}x + 1 \end{cases}$



5. $\begin{cases} y \leq \dfrac{1}{2}x + 2 \\ y > \dfrac{-2}{3}x + 1 \end{cases}$



ASSESSMENT

Consider the equation $\begin{cases} x - y \leq 2 \\ x < y \end{cases}$ and $\begin{cases} y > x + 1 \\ y < x \end{cases}$ and ask students to find the solution, (how many solution does each system of inequality have? Why?, in more than one way (multiple representation of the solution, including guess, check and graphical method). The significance of representing the solution of linear equations in multiple ways provides the same objective of more than one form. You can assess whenever necessary.

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving linear expression, and/or tests/quizzes.

3.3. Inequalities Involving Absolute Value

Periods allotted 3

Competency

- solve inequalities involving absolute value of linear expression in one variable.

Key words: Inequalities, Absolute value.

The following are the general rules to consider when solving absolute value inequalities:

- Isolate on the left side of the absolute value expression.
- The type of inequality sign determines the format of the compound inequality to be formed.
- Solve the positive and negative versions of the absolute value inequality.
- When the number on the other side of the inequality sign is negative, you either conclude all real numbers as the solutions, or the inequality has no solution.

  Consider $|2x - 4| > -3$ and $|2x - 4| < -3$. Find the solutions for each. The first has all real numbers as a solution and the second has no solution.
- When the number on the other side is positive, we proceed by setting up a compound inequality by removing the absolute value bars.
  Consider $|2x - 4| > 3$ and $|2x - 4| < 3$. For the first inequality, $2x - 4 > 3$ *or* $2x - 4 < -3$. For the second inequality, $-3 < 2x - 4 < 3$.

Answers to activity 3.3

The above red colored on the number line represents $-7 < x < -1$

implies $-7 + 4 < x + 4 < -1 + 4$

$-3 < x + 4 < 3$ which in turn represents the absolute value inequality $|x + 4| < 3$. You try to show your students the remaining different types of inequalities ($\leq$, $>$, $\geq$) using different absolute value inequality.

Answers to Exercise 3.8

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

a. $-10 < x < 4$

b. The set of all real numbers

c. Empty set

d. $x > \frac{7}{12}$ or $x < \frac{5}{12}$

e. $-7 < x < 7$

f. $x < -6 \ or \ x > 2$

ASSESSMENT

Consider the absolute value inequality $|4x + 2| \geq 0$, $|x - 2| < 10$ and $|4x - 2| \leq -3$ and $|-x + 2| > 8$ and ask students to find the solution in more than one way as assessments for learning. Furthermore, you can ask the number of solutions.

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving linear expression, and/or tests/quizzes.

3.4 Quadratic Inequalities

Periods allotted 9

Competency

- solve quadratic inequalities by using product properties.
- solve quadratic inequalities using the sign chart method.
- solve quadratic inequalities using graphs.

Key words: Inequalities, Quadratic.

To solve a quadratic inequality, you apply the method as illustrated in the procedure below:

- Write the quadratic inequality in standard form: $ax^2 + bx + c$ where $a, b\ and\ c$ are coefficients and $a \neq 0$.
- Determine the roots of the inequality.
- You can use case method or product method or graphical method.
- Write the solution in inequality notation or interval notation.

Consider the following as an example.

In solving $(x+2)(x-4) \geq 0$,

Separate into possible cases:

Case 1: $\begin{cases} x+2 \geq 0 \\ x-4 \geq 0 \end{cases}$

$x \geq -2$ & $x \geq 4$. The intersection: $x \geq 4$. or

Case 2: $\begin{cases} x+2 \leq 0 \\ x-4 \leq 0 \end{cases}$,

$x \leq -2$ & $x \leq 4$. The intersection: $x \leq -2$

3.4.1 Solving quadratic inequalities using the product property (3 periods)

Competency

- solve quadratic inequalities by using product properties.

Answers to Activity 3.4

1. a) If the signs of both numbers are the same, then the product will be positive.

b) If the signs of the numbers are the different, then the product will be negative.

c) If one of the numbers is zero, then the product is zero.

2. a) $(x-4)(x-4)$ **b)** $(x-3)(x+2)$ **c)** has no factors in real numbers

d) $(3x-2)(x-5)$

Answers to Exercises 3.9

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. $2 < x < 4$
2. $x > 2 \text{ or } x < -2$
3. $-3 \leq x \leq 5/2$
4. All real numbers
5. $-3/2 \leq x \leq 4$
6. Empty set or no solution

3.4.2. Solving quadratic inequalities using sign chart

(3 periods)

Competency

- solve quadratic inequalities using the sign chart method.
- Solving quadratic inequalities using sign chart is using product properties with the help of a sign chart in a table or in a number line. You can employ class activity that you used in activity 3.4 in table form so that you can show inequalities in sign chart.

Answers to Exercises 3.10

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. $-1 < x < 3$
2. $All\ real\ numbers$
3. $x < -\frac{4}{3} or\ x > 1/2$
4. $2/5 \leq x \leq 3/2$
5. $-5 \leq x \leq 1$
6. $x < -1\ or\ x > 1/2$
7. $x < -2\ or\ x > 3$
8. $No\ solution$

ASSESSMENT

Consider the equation $x^2 + 2x + 1 \geq 0$, $x^2 + 5x + 6 \geq 0$, $3x^2 + 12x + 8 > 0$ and $4x^2 + 2x + 1 < 0$ and ask students to find the solution in more than one way(product property method, sign chart method). Ask also the number of solutions. You can assess whenever necessary.

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercise problems on solving equations involving linear expression, and/or tests/quizzes.

3.5 Applications on Inequalities

Periods allotted 2

Competency

- Apply inequalities in real life situations.

Key words: Inequalities, Applications.

In Activity 3.6, let the students translate real-life situations into linear inequalities in two variables. Give emphasis on the meanings of the phrases "less than", "more than", "greater than", "at most" and "at least". Let the students differentiate also "less than" and "is less than" and "more than" and "is more than". Provide examples on how these are used for students to understand their differences.

Answers to activity 3.6

a. $10\mathrm{b} + 4\mathrm{p} > 180$

b. $\mathrm{A} - \mathrm{M} \geq 10$

c. $5\mathrm{r} - 5 < \mathrm{h}$

d. $\mathrm{f} + \mathrm{e} + \mathrm{r} \leq 5{,}000$

Let students broaden their understanding of linear inequalities in two variables as to how they are used in solving real-life problems. Ask them to perform Activity 3.7. Encourage them to use different ways of arriving at the solutions to the problems. More importantly, provide them the opportunities to choose the most convenient way of solving each problem.

Answers to activity 3.7

a) $5r + 2c < 600$ **b)** $5(35) + 2c < 600$. Which gives $c < 212.5 \cong 213$. The greatest cost of a kilo of coffee is approximately $Birr\ 213$

c) $5(34) + 2c > 600$. Hence, $c > 215$. Meskerem has to pay at least Birr 215 for 2 kilos of coffee.

Answers to Exercise 3.11

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1.

- **a.** $0 < x - 6 < 8$ which implies that $6 < x < 14$. Or $0 < 6 - x < 8$ which implies that $-2 < x < 6$
- **b.** $2 < 2x < 12$. Hence, $1 < x < 6$
- **c.** $1 + 2x > 5 \; or \; 1 + 2x < 1$. Therefore, $x > 2 \; or \; x < 0$
- **d.** $\frac{1}{3}x < 2 \; or \; \frac{1}{3}x > 5$. Therefore, $x < 6 \; or \; x > 15$

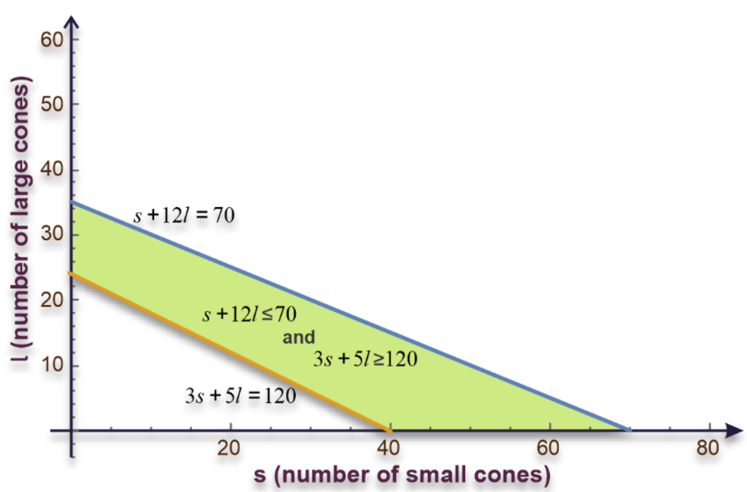
2.

- **a.** $4{,}800 \leq Number\ of\ White\ blood\ cells\ per\ cubic\ millimeter \leq 10{,}800$
- **b.** $Number\ of\ White\ blood\ cells\ per\ cubic\ millimeter < 4{,}800\ or$
  $Number\ of\ White\ blood\ cells\ per\ cubic\ millimeter > 10{,}800$

3. First, identify the variables. There are two variables: The number of small cones and the number of large cones.

$$s = small \quad cone$$
$$l = large \quad cone$$

The first equation is $s + 2l \leq 70$ and the second equation is $3s + 5l \geq 120$. Solve simultaneously by graphing both on the same x-y axis.



The region in purple is the solution. As long as the combination of small cones and large cones that Almaz sells can be mapped in the purple region, she will have earned at least Birr 120 and not used more than 70 scoops of sambusa.

4. Let x be the allowable speed. Then, $45 \le x \le 55$

5. Let $x = the\ actual\ diameter$

   $|x - 88.00| \le 0.007$. Solving the absolute value, we have the following:

   The range of acceptable diameter for the piston is $87.9993mm \le x \le 88.007$

6. Let x be the allowed speed. The absolute value inequality that models this situation is $|x - 65| < 9$. The speed you are allowed to go without getting a ticket is the solution of $|x - 65| < 9$. That is $56 < x < 74$

7. Let $x =$ the longer leg

   $x - 7 =$ the shorter leg

   $x^2 + (x-7)^2 \ge 13^2 \Rightarrow x^2 - 7x - 60 \ge 0$

   Solve the quadratic equation $x^2 - 7x - 60 = 0$

   $\Rightarrow x = 12,\ \&\ \ x = -5$. However, $x = -5$ can never be a solution. Why?

   Therefore, the shorter leg should be at least $x - 7 = 12 - 7 = 5$.

8. Let $x =$ the width

   $x + 3$ will be the length of the plot

   $x(x+3) \le 18\ implies\ x^2 + 3x - 18 \le 0$

   Solving the quadratic inequality gives us that the dimensions should be at most 3 by 6

ASSESSMENT

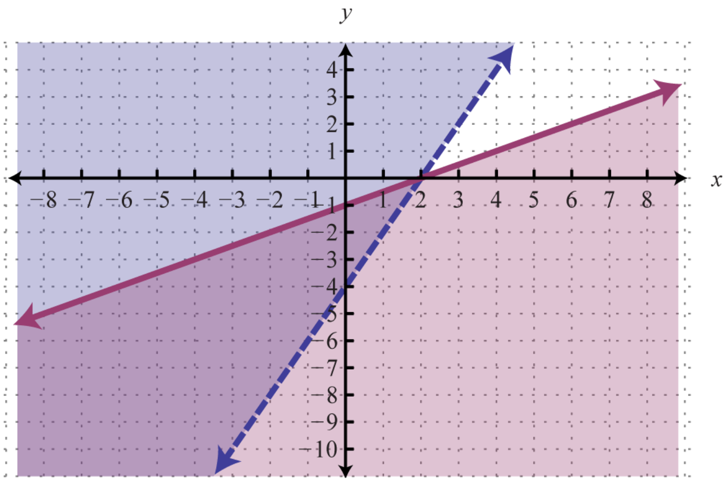
Consider the following question: Alemitu needs to make a rectangular plot which has a perimeter of at most $100m$. The length should be $20m$ longer than the width. What are the approximate possible dimensions of the plot?

Ansewrs to Review Exercise on Unit 3

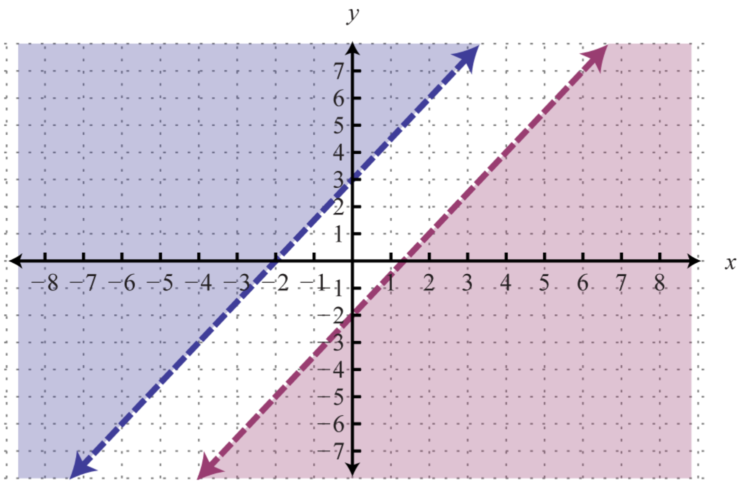
Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. If $x > y$, then $ax > ay\ for\ a > 0.\ i.e\ the\ inequality\ sign\ never\ changes$, if multiplied by a, then the inequality sign will be changed to " = " and if $x > y$, then $ax < ay\ for\ a < 0$
2. **a)** $\{1,2\}$
3. **a)** Let $x\ be\ the\ number, then\ \ 7 + 3x \leq 1$

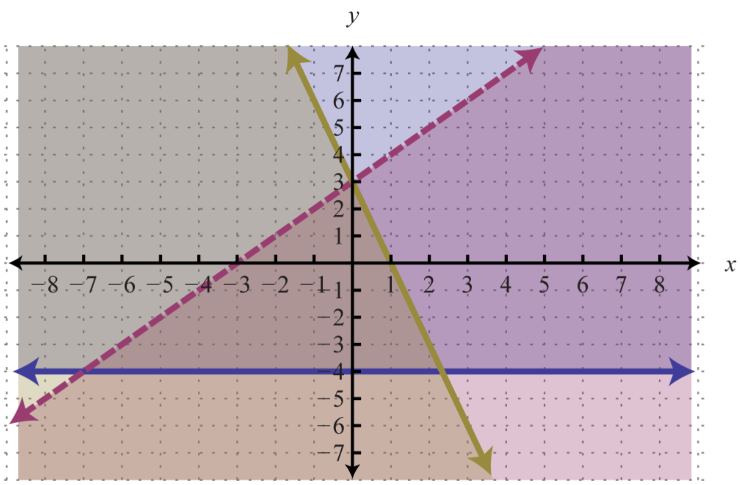
   b) Let $x\ be\ the\ number, then\ \ 2y - 3 \geq 9$
4. **a.** $-3 < x < 3$ **b.** $\frac{5}{3} < x < 4$ **c.** $4 \leq x < 7$
5. **a.** First find the intersection of the two lines i.e, $(2,0)$.



   b. No solution as there is no common intersection region.



c.



6. **a.** $-\frac{1}{2} \leq x \leq \frac{3}{2}$, **b.** All real numbers **c.** No solution **d.** $-\frac{3}{2} \leq x \leq \frac{21}{2}$

e. $x < -\frac{5}{3}$ or $x > \frac{7}{3}$ **f.** $2 \leq x \leq 4$, **g.** $0.0034 \leq i \leq 0.0038$ **h.** No solution

7. **a.** $x < 1$ or $x > 2$ **b.** $-3 \leq x \leq 5$ **c.** $x < -3$ or $x > 1$ **d.** $x < -8$ or $x > 0$

e. No solution **f.** All real numbers **g.** $-2 \leq x \leq 6$ **h.** $(-\infty, 0) \cup (3, \infty)$

8. $x > \sqrt{3}$ or $x \leq -\sqrt{3}$ **a.** $r = 1$ and $r = -\frac{1}{9}$ **b.** $-\frac{1}{9} \leq r \leq r$ **c.** No solution

d. All real numbers.

9. D **10.** D **11.** A **12.** C

Unit 4

INTRODUCTION TO TRIGONOMETRY

(7 periods)

Introduction

The main task of this unit is to introduce students about trigonometric ratios of sine, cosine and tangent of some special angles ($0°, 30°, 45°, 60°$ and $90°$). The unit is subdivided into two subunits. The topics dealt with in the unit are revision on right-angled triangle and trigonometric ratios.

Trigonometry was originally developed to solve problems related to astronomy, but soon found applications to navigation and a wide range of other areas. It is of great practical importance to builders, architects, surveyors and engineers and has many other applications.

Suppose we lean a ladder against a vertical wall. By moving the ladder closer to the wall, thereby increasing the angle between the ladder and the ground, we increase the distance up the wall that the ladder can reach. Since the length of the ladder remains the same, Pythagoras' theorem relates the distance up the wall to the distance of the ladder from the base of the wall. Trigonometry allows us to relate that same distance to the angle between the ladder and the ground.

Unit Learning Outcomes

After completing the unit, students will be able to:

- define sine, cosine, and tangents ratios
- find trigonometric values of angles from trigonometric table

Suggested teaching aids for the unit

In addition to the student's textbook and the teacher's guide, you are advised to prepare and bring into the class the following materials whenever the topic requires.

Teaching aids: pair of compass, protractor, scissors, thin card boards, colored markers, drawing papers, Calculator, cm rulers. It is also possible to use various mathematical software such as Geometer's Sketchpad, Mathematica, geogebra, etc. whenever they are available.

4.1 Revision on Right-angled Triangle

Periods allotted: 2 periods

Competencies

At the end of this subunit, students will be able to:

- define a right-angled triangle
- identify the hypotenuse, opposite and adjacent of a right-angled triangle
- describe the basic properties of a right-angled triangle

Introduction

This sub-unit deals with revising right angled triangle with its properties. Students have already studied about triangles, and in particular, right triangles, in their earlier grades. So to begin, it is better to motivate the students by giving an insight of the unit. Ask students to list triangles in particular right-angled triangle models of their environment. Following these discussion you can formally start the topic.

Possible teaching aids for the subtopic

Use ruler and protractor to construct different right angle triangles.

Right triangle and Pythagoras theorem **Periods allotted: 1 period**

Answer for Exercise 4.1

1.

a. $16^2 + x^2 = 19^2$, solving for x, $x^2 = 19^2 - 16^2 = 361 - 256 = 105$ so that $x = \sqrt{105}$

b. $x^2 + x^2 = 8^2$, solving for x we have $2x^2 = 64$ hence $x = 4\sqrt{2}$

c. $5^2 + x^2 = 10^2$, solving for x we get $x^2 = 10^2 - 5^2 = 100 - 25 = 75$, $x = 5\sqrt{3}$

2.

a. AC is a perpendicular bisector for an isosceles triangle ABD so that $AC = 12$. Hence using right angled triangle ABC, we use Pythagoras theorem. That is: $12^2 + x^2 = 13^2$,

$x = \sqrt{13^2 - 12^2} = \sqrt{25} = 5$

b. $AB^2 + x^2 = 3^2$. Since DAC is an isosceles triangle BD bisects AC so that $AB = 2$.

Hence, $x^2 = 9 - 4$ results $x = \sqrt{5}$.

c. Using $2^2 + AC^2 = (2\sqrt{5})^2$, $AC = 4$. Again for a $AC^2 + CD^2 = AD^2$,we get $x = \sqrt{40} = 2\sqrt{10}$.

Conversion of the Pythagoras theorem

Periods allotted: 1 period

Answer for Exercise 4.2

1.

a. The possible candidate to be the legs of a triangle be the smallest two sides, 4 and 6 and the hypotenuse may be 8. Let us check it using Pythagoras theorem.
$4^2 + 6^2 = 52$, compare it with $8^2 = 64$. $52 \neq 64$. So that the given sides cannot be sides of a right-angled triangle.

b. $\sqrt{2}^2 + \sqrt{3}^2 = \sqrt{5}^2$, so that $\sqrt{2}, \sqrt{3}$ *and* $\sqrt{5}$ are sides of a right angled triangle.

c. $1, 2, \sqrt{3}$ are sides of a right angled triangle since $1^2 + \left(\sqrt{3}\right)^2 = 2^2$.

d. $\sqrt{3}^2 + \sqrt{6}^2 = 9 = 3^2$, so that $\sqrt{3}, \sqrt{6}$ *and* 3 are sides of a right angled triangle.

2.

a. To check whether a triangle is a right angled triangle or not. The largest side is 28 and the other two sides are 19 and 20. Let us test whether $19^2 + 20^2$ is equal to 28^2 or not. Observe that $761 \neq 784$. Hence the triangle is not right- angled triangle.

b. The largest side is 25 and the other two sides are 24 and 8. Let us test whether $8^2 + 24^2$ is equal to 25^2 or not. Observe that $640 \neq 625$. Hence the triangle is not right- angled triangle.

c. The largest side is 65 and the other two sides are 33 and 56. Let us test whether $33^2 + 56^2$ is equal to 65^2 or not. Observe that $4{,}225 = 4{,}225$. Hence the triangle is right angled-triangle.

Assessment

You can assess the students by drawing different triangles on graph paper and ask them to identify the right angled-triangle either by applying Pythagoras theorem or measuring angle using protractor.

4.2 Trigonometric Ratios

Periods allotted: 5 periods

Competencies

At the end of this sub unit students will be able to

- find trigonometric values of angles from trigonometric table.
- solve word problems related to trigonometric ratios.
- determine sine, cosine and tangent of angles between 0°and 90°from a trigonometric table.

Introduction

The main objective of this subunit going to be about trigonometric ratios and problem solving with trigonometry. The definitions of sine, cosine and tangent for some special acute angles $(0°, 30°, 45°, 60°\ and\ 90°)$ to be determined using Pythagoras theorem. Students can use trigonometry to enrich their mind and expand their thinking, but more practically trigonometry can be used in surveying, engineering problems, stock market trends or may be business cycles. Furthermore, needs the basics of trigonometry to apply for the next units and other subjects like Physics.

At this level you will focus on the trigonometric ratios of sine, cosine and tangent for the above special angles. Reading values from a trigonometric table for angle (θ) between $0° \leq \theta \leq 90°$ will be discussed. You also design a lesson to determine lengths (height) of their surrounding using the trigonometric ratio. For example you may give field work to students to determine the height of the flag pole of their school using trigonometry.

You can start the lesson by asking oral questions about like 'Is it possible to determine the height of Mountain Ras Dashen without measuring it?' or similar question and use the questions and graphs available on the students' textbook. Then define the trigonometric ratio and support with examples.

Possible teaching aids for the subtopic

Use ruler and protractor to construct different right angle triangles. Use different colored chalk for the acute angles. If the school has a computer lab, you can use geogebra software to check to see length of sides of a triangle and measure of angles of a triangle.

Trigonometric ratios

Periods allotted: 1 period

Answers for Exercise 4.3

From the given figure, we can apply the definition and determine the trigonometric ratios of sine, cosine, and tangent as follows

a) $\sin A = \frac{12}{13}$, $\cos A = \frac{5}{13}$ *and* $\tan A = \frac{12}{5}$.

b) $\sin A = \frac{6}{10}$, $\cos A = \frac{8}{10}$ *and* $\tan A = \frac{6}{8}$.

c) $\sin A = \frac{1}{\sqrt{5}}$, $\cos A = \frac{2}{\sqrt{5}}$ *and* $\tan A = \frac{1}{2}$.

Answers for Exercise 4.4

1. a. We have right angled triangle of the following type. Given that $\sin A = \frac{1}{2}$, this is to mean the side ratio of BC to the hypotenuse AC is 1:2,so using Pythagoras theorem,

$AB^2 + BC^2 = AC^2$. It results $AB = \sqrt{2^2 - 1^2} = \sqrt{3}$.

Hence, $\cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$ and $\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$.

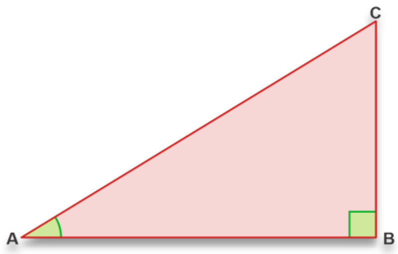


Figure 4.1

b. You can follow similar procedure to that of (a).

Given that $\cos A = \frac{2}{3}$, that is the ratio of sides AB to AC is 2:3,

so that using Pythagoras theorem we can determine BC.

$BC = \sqrt{3^2 - 2^2} = \sqrt{5}$.

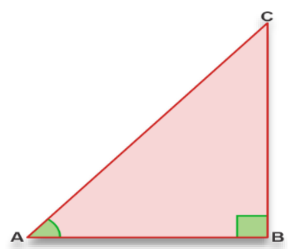


Figure 4.2

Hence, $\sin A = \frac{BC}{AC} = \frac{\sqrt{5}}{3}$ and $\tan A = \frac{BC}{AB} = \frac{\sqrt{5}}{2}$

c. You can construct similar right angled triangles and get the length of the hypotenuse is $3\sqrt{5}$.

Hence, $\sin A = \frac{1}{\sqrt{5}}$ *and* $\cos A = \frac{6}{3\sqrt{5}} = \frac{2\sqrt{5}}{5}$.

d. $\cos A = \frac{2}{\sqrt{5}}$ *and* $\tan A = \frac{1}{2}$.

Trigonometric values of basic angles

Periods allotted: 1 period

Answers for Exercise 4.5

1.

	$\angle A = 30°$	$\angle A = 45°$	$\angle A = 60°$
$\sin A$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos A$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan A$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

2. **a.** The triangle is an isosceles right triangle and so that $= y$. $x^2 + x^2 = (13\sqrt{2})^2$, solving for , we get $x = y = 13$.

b. Similar to (a) above we obtain $y = \sqrt{14}$

c. Using trigonometric ratio for 30° or 60°, we obtain $x = 10$ and $y = 10\sqrt{3}$.

d. Using trigonometric ratio for 30° or 60°, we obtain $x = 16$ and $y = 8\sqrt{3}$.

e. Using trigonometric ratio for 30° or 60°, we obtain $x = 10$ and $y = 20$.

f. We can determine a and c, we use the smaller triangle which is an isosceles right triangle. Here, $a = c$, $a^2 + a^2 = (5\sqrt{2})^2$. Solving for a we get $a = c = 5$. Using this value on the second triangle , we obtain $b = 10$ and $d = 5\sqrt{3}$.

g. $BC = 12$ and $AB = 10$ since $ABCE$ is a rectangle. Using trigonometric ratio for 30° or 60° , we obtain $CD = 10$ and $ED = 10\sqrt{2}$.

Trigonometric Ratios of 0^0 and 90^0

To begin the next lesson, you might ask oral questions whether students understood how to determine the trigonometric ratios of $30°, 45°$ *and* $60°$. Then after tell them the trigonometric ratios of 0° and 90° without any derivation.

If an outstanding student needs the derivation here is the approach of determining trigonometric ratios of 0° and 90°.

Let us also see what happens to the trigonometric ratios of angle A, if it is made smaller and smaller in the right angle triangle ABC. As $\angle A$ gets smaller and smaller, the length of the side BC decreases.

The point C gets closer to point B, and finally when $\angle A$ becomes very close to 0°, AC becomes almost the same as AB (see fig 4.3).

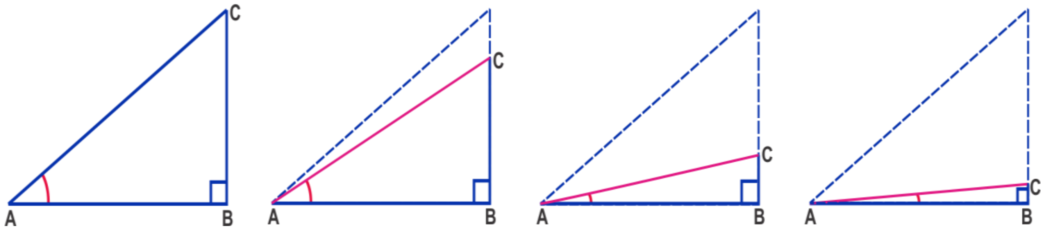


Figure 4.3

When $\angle A$ very close to $0°$,BC gets very close to 0, so that, the value of $\sin A = \frac{BC}{AC}$ is very close to 0. Also when $\angle A$ is very close to 0°, AC is nearly the same as AB so the value of $\cos A = \frac{AB}{AC}$ is very close to 1. This helps us to see how we can define the values of $\sin A$ and $\cos A$ when the measure of $\angle A$ is equal to zero degree. We define $\sin 0° = 0$ and $\cos 0° = 1$. Using these, we have $\tan 0° = \frac{\sin 0°}{\cos 0°} = 0$.

Now, let us see what happens to the trigonometric ratios of $\angle A$, when it is made larger and larger in ΔABC till it becomes 90°. As $\angle A$ gets larger and larger, $\angle C$ gets smaller and smaller. Therefore, as in the case above, the length of the side AB goes on decreasing. The point A gets closer to point B. Finally when $\angle A$ is very close to 90°, $\angle C$ becomes very close to 0° and the side AC almost coincides with side BC (see figure 4.4 below).

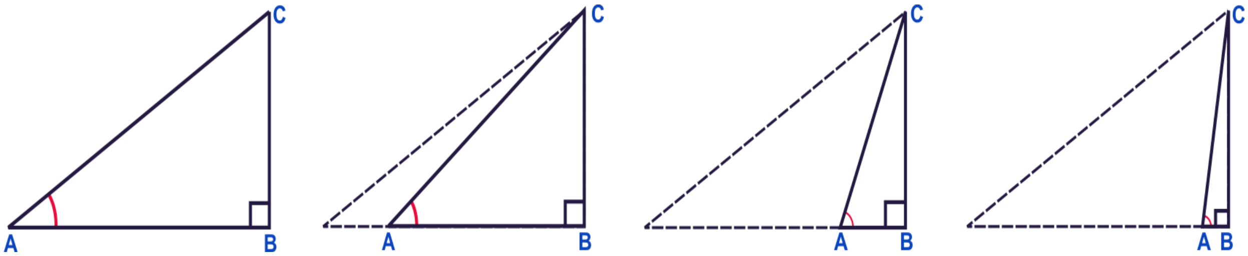


Figure 4.4

When $\angle C$ is very close to 0°, $\angle A$ is very close to 90°, side AC is nearly the same as side BC, and so $\sin A$ is very close to 1. Also when $\angle A$ is very close to 90°, $\angle C$ is very close to 0°, and the side AB is nearly zero, so $\cos A$ is very close to 0.

So, we define: $\sin 90° = 1$, $\cos 90° = 0$ and $\tan 90°$ is undefined (apply definition of trigonometric ratio).

At this level focus on the value rather than how derive it on the trigonometric ratios for $0°$ and $90°$.

Trigonometrical values of angles from a table

Periods allotted: 1 period

Answers for Exercise 4.6

a. 0.390731 **b.** 0.374606 **c.** 7.115376

Determining angle from a table for a given value

Periods allotted: 1 period

Answers for Exercise 4.7

1. **a.** $77°$ **b.** $10°$ **c.** $50°$ **d.** $64°$

2. The base is 8 cm and the height is 5cm, so to determine the angle between the hypotenuse and the base say θ we use $\tan\theta = \frac{5\text{ cm}}{8\text{ cm}} = 0.625$ therefore, $\theta = 32°$.

Revision and application problem

Periods allotted: 1 period

Students have no previous experience regarding to trigonometric ratios. So it is better to revise the unit and give at least one or two application problem. These problems can be taken either from the review exercise or from text books or materials.

Assessment

You can assess the students by providing different acute angles for students and determine sine cosine and tangent from the trigonometric table. You can also give more applicable problems related to their environment besides given on the text book.

Answer for Review Exercises on Unit Four

1. **a.** False (it could also greater than 1).

b. False (the ratio could not be greater than 1)

c. True

d. True

2. In the given triangle ΔABC, right angled at B as shown in figure 4.5 below. Given that

$AB = 24\ cm, BC = 7cm$,. Using Pythagoras theorem $= \sqrt{(24)^2 + 7^2} = 25$. Now we are

asked to determine

a. $\sin A = \frac{7}{25}$, $cosA = \frac{24}{25}$,

b. $\sin c = \frac{24}{25}$, $\cos c = \frac{7}{25}$

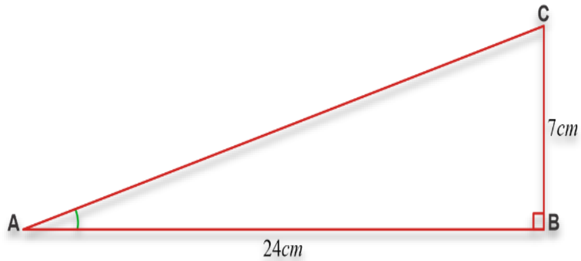


Figure 4.5

3. **a.** 0.309017 **b.** 0.544639 **c.** 2.904214

4. All A, B and C are properties of right-angled triangles. For acute angle θ the value of $\cos\theta$ is between 0 and 1. Hence, the answer is D.

5. $\tan\theta = \frac{\sin\theta}{\cos\theta} = 1$ implies $\theta = 45°$. Answer is D, since $\tan\theta \neq \sin\theta$.

6. Let us consider a right angle triangle ABC as shown in Figure 4.6 below which satisfy the condition given in the problem. Using Pythagoras theorem $AC = \sqrt{5}$ unit. Hence, $\cos\theta° = \frac{\sqrt{5}}{3}$ and $\tan\theta° = \frac{2}{\sqrt{5}}$. The other acute angle of this triangle that is $m(\angle B) = 90° - \theta°$. For this angle, the adjacent side has length 2 unit and the hypotenuse is 3 unit.So that $\cos(90° - \theta°) = \frac{2}{3}$. Also if we compare the two values, $\sin\theta° = \frac{2}{3}$ and $\cos\theta° = \frac{\sqrt{5}}{3}$. $\sin\theta° < \cos\theta°$. Hence from above alternatives the correct one answer is B

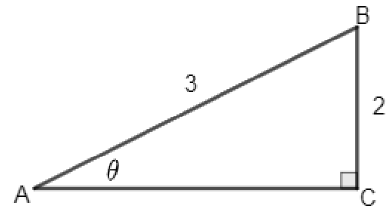


Figure 4.6

7. Let the tower be CD and the two women be located at A and B respectively as shown in figure 4.5 below. Here. $AB \perp CD$ and let $AC = x$ and $CB = y$. What we need to determine is $x + y$.

Using trigonometric ratio,

$\tan 60° = \frac{150}{x}$ and $\tan 30° = \frac{150}{y}$.

So that $x = \frac{150}{\tan 60°}$, that is $x = 50\sqrt{3}$ m

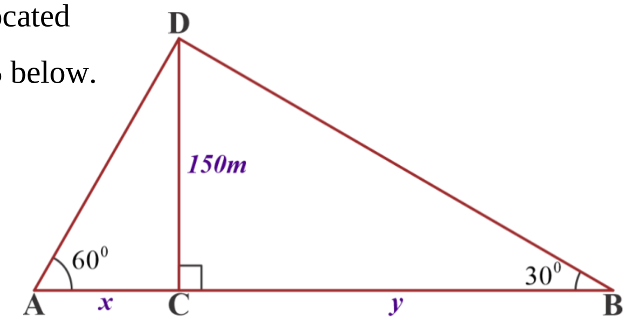


Figure 4.7

and $y = \frac{150}{\tan 30°}$, that is $y = 150\sqrt{3}$ m. So that, $x + y = 200\sqrt{3}$ m. Hence the answer is C.

8. You can search from the table, the approximate degree is 85°.

9. The angle of elevation is 60° (the observer is at the bottom and the angle is measured from the horizontal to the line of observation. We immediately get

$\tan 60° = \frac{h}{50m}$. Hence,

$\mathrm{h} = \tan 60° \times 50m$

$h = 50\sqrt{3}$ m.

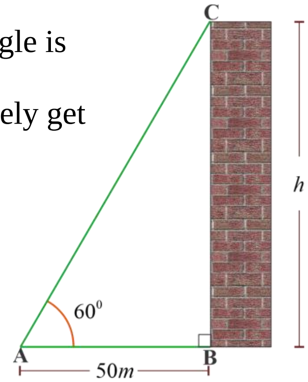


Figure 4.8

10. The figure 4.9 shows the top of the building P.The boat S and the base of the building B. Let $SB = x\ m$ be the distance of the boat from the building. By alternating angles $\angle BSP = 25°$.

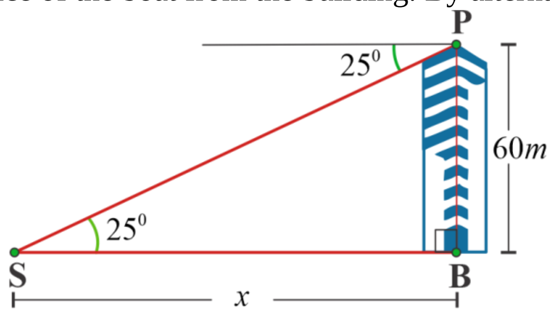


Figure 4.9

a. Hence, using the trigonometric ratio of tangent we can determine x as follows

$$\tan 25° = \frac{PB}{SB} = \frac{60m}{xm} \text{ so that } x = \frac{60}{\tan 25°} \approx 129 \text{ m}$$

b. If SB is determined we can able to get the distance between P and S using Pythagoras theorem.

$$\text{That is,} PS = \sqrt{(SB)^2 + (PB)^2} = \sqrt{(129)^2 + (60)^2} \approx 142 \text{ m}$$

11. The height of the building is $H = h_1 + h_2$. Given that

$h_1 = 1.74m$, $\theta = 63°$ and the distance from the man to the building $100m$.

Now let us determine h_2.

From the right angled triangle ABC, $\tan 63° = \frac{h_2}{100m}$.

So that $h_2 = 100m \times \tan 63° = 196.26m$.

Hence, the height of the building is

$H = h_1 + h_2 = 1.74m + 196.26m = 198m$.

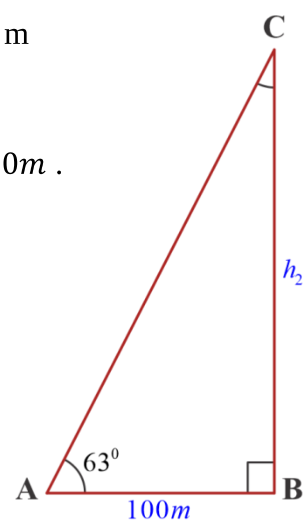


Figure 4.8

Unit 5

REGULAR POLYGONS (12 periods)

Introduction

In this unit, the basic geometric notions learned in lower grades are strengthened. Students had an experience about triangles, quadrilaterals and other polygons. Hence, those concepts will be revised before directly approaching regular polygons. Students should recapitulate the definition of parallelogram, rectangle, rhombus, square and trapezium. Students will discover various patterns involving the sum of the measure of interior and exterior angles of regular polygons, and use these patterns to develop a set of rules. Finally, they should be able to derive area and perimeter formulae for regular polygons.

Unit Learning Outcomes

After completing the unit, students will be able to:

- identify regular polygons
- calculate area of regular polygons
- calculate perimeter of regular polygons.
- find the measure of each interior or exterior angle of a regular polygon
- understand properties of regular polygons.

Suggested Teaching Aid for unit 5

This unit deals geometric figures so that different teaching aids should be used in order to the concepts through visualization. So the following are some of the materials needed for the teaching learning process of this unit

Colored chalks, markers, graphical papers, ruler, compass and protractor, model of triangles and other polygons.

Order students to construct models of regular polygon as a group work. While doing such group work check the participation of each student either by following their activity or asking different verifying questions . It is also advisable to show these regular polygons using mathematical software like Geogebra, Matlab or any related software found in the school computer laboratory.

5.1 Sum of Interior Angles of Convex Polygons

Periods allotted: 3 periods

Competencies

At the end of this subunit, students will be able to:

- formulate conjectures about interior angles of polygons.
- explore the relationship of the number of sides to its interior angles of polygons.
- define interior angles of polygons.
- identify concave and convex polygons.

Introduction

Students have learned about triangles, quadrilaterals, and other polygons in lower grades. You need to provide different activities to recall the different properties of these integrals. Specially give stress to make students be familiar on the sum the measure of interior angles of a triangle is 180° . Because this is the basics for the subsequent subunits. Ask oral questions frequently to check your students follow up at each lesson.

The approach to deliver the lesson first giving activities which lead to the definition or certain conclusion and then provide appropriate example.

The main objective of this subunit is to arrive at the following conclusion.

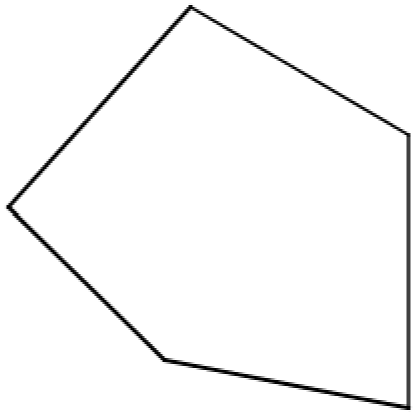
Sum of the measure of interior angles of n sided polygon is equal to $(n-2)180^0$.

You will start from a triangle and reach to every n- gon polygon. Start the lesson by giving activity 5.1.

Answer to Activity 5.1

1. A triangle is a polygon with 3 sides and 3 vertices.
2. A quadrilateral is a 4 sided polygon with 4 vertices.

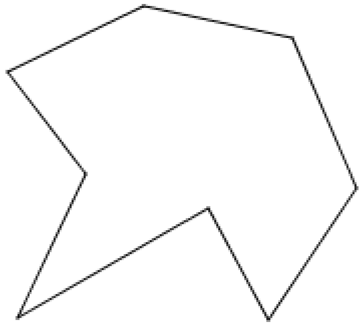
   Example:- square, rectangle, rhombus, trapezium
3. A plane figure that has 5 sides and corners



Answer for Exercise 5.1

1. 9 **2.** Decagon

3. There are many options, one of them could be the following



4. The remaining polygon is a triangle

Answer for Exercise 5.2

1. Concave

2. **a.** heptagon, concave
b. decagon, convex
c. octagon, convex
d. heptagon, concave
e. 11-sided polygon, concave
f. nonagon, concave

Answer for Activity 5.2

From their lower grade mathematics some of them may try to respond the following, if they are not able to reflect it, tell the student you will answer these questions after the following lessons.

1. 180°

2. 360°
3. Yes
4. By dividing into a number of triangles using a fixed vertex.

Answer for Exercise 5.3

1. 5
2. For hexagon, $n = 6$ so that the sum of the measure of interior angles is $(n-2) \times 180°$, so that the sum of interior angles of a hexagon is $4 \times 180° = 720°$.
   Similarly, for heptagon ($n = 7$), the sum of interior angles of a hexagon is $5 \times 180° = 900°$.

Answer to Exercise 5.4

1. We use a formula which states as the sum of interior angles of n-gon polygon is $(n-2) \times 180^0$.
   a. For $n = 7$, the answer is $900°$.
   b. For $n = 9$, the answer is $1{,}260°$.
   c. For $n = 12$, the answer is $1800°$
2. Given that $(n-2) \times 180^0 = 1{,}440°$, solving for n ,we get $n = 10$.
3.

Polygon	Measure of 1st angle	Measure of 2nd angle	Measure of 3rd angle	Measure of 4th angle	Measure of 5thangle	Measure of 6th angle
Triangle	34°	96°	50°			
Quadrilateral	100°	45°	105°	110°		
Pentagon	80°	105°	90°	80°	185°	
Hexagon	110°	190°	140°	178°	62°	40°

4. The pentagon and the hexagon are concave whereas the triangle and a quadrilateral are convex polygons.

Assessment

You can assess students by giving different polygons made from hard paper and asking them to check the sum of interior angles of a polygon. This could be done in groups and they can reflect after they get an answer to the class.

5.2. Sum of Exterior Angles of Convex Polygons

Periods allotted: 2 periods

Competencies

At the end of this subunit, students will be able to:

- define exterior angles of a convex polygon
- determine the sum exterior angles of a polygon

Introduction

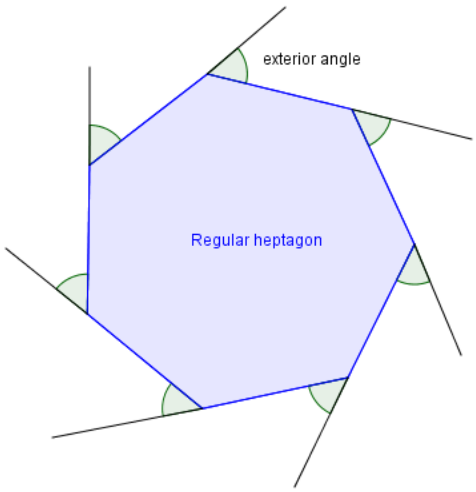
In subsection 5.1 you discussed how to determine the sum of interior angles of a polygon. In this subsection you will present lessons to define and show the procedure to reach a conclusion of determining the sum of exterior angles of a polygon. You facilitate the lesson by asking oral questions, leading the activity and making the class active. Give much time to students to discuss each other when they try to derive the formula. At last the main students reach to a conclusion that the sum of exterior angles of a polygon is 360°.

Answer for Activity 5.3

It is given just to initiate students to know about exterior angles of a polygon. It will be discussed after defining exterior angles of a triangle which is similar to figure 5.7 on the student textbook.

Answer for Exercise 5.5

1.



2. **a.** False **b.** True **c.** True

Answer for Exercise 5.6

The sum of the measure of exterior angles of a hexagon is $360°$.

Answer for Exercise 5.7

1. First determine the measure of the 5th remaining angle (say, x) of the pentagon. We know that the sum of the interior angles of a pentagon is $(n-2) \times 180 = 3 \times 180° = 540°$. Subtract the sum of the measure of these 4 angles from $540°$. Hence, we get the remaining angle measure is equal to $155°$.

Now the measure of each exterior angle corresponding to the interior angles $95°, 100°, 120°, 70°$ *and* $155°$ is $85°, 80°, 60°, 110°$ *and* $25°$ respectively.

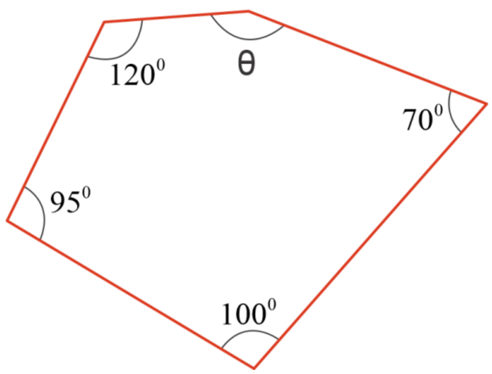


Figure 5.1

2. From the figure 5.2 below , we will find the value of $x°$ and $y°$ using, the idea of sum of interior angles of a triangle, the supplementary angle to $160°$ is $20°$ .The supplementary angle of $y°$ is $x°$. Hence, $160° = 2x°$, that is $x° = 80°$, so that $y° = 100°$

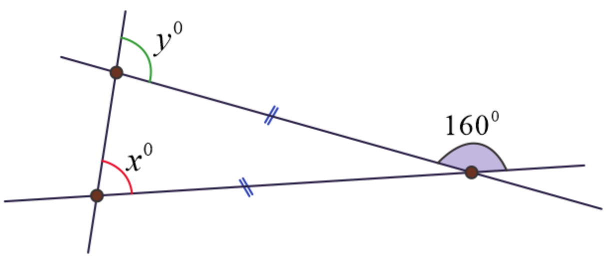


Figure 5.2

5.3 Measures of each Interior Angle and Exterior Angle of a Regular Polygon

Periods allotted:2 periods

Competencies

At the end of this subunit, students will be able to:

- define a regular polygon
- determine each interior and exterior angle of a regular polygon

Introduction

In the previous two subunits students have studied how to get the sum of the measure of interior and exterior angles of n-sided polygon. In this subunit you will define formally regular polygon after activity 5.4. Following the definition, you can help students to list some regular polygons from their environment, like the honeycomb mentioned in the students textbook. Finally lead the students to derive a formula to determine the measure of each interior and exterior angles of a regular polygon.

Answer for Activity 5.4

The given polygons have congruent sides and congruent interior angles.

Answer for Exercise 5.8

1. **i)** The regular hexagon has 6 sides so that each interior angle measures $\frac{(6-2)\times180°}{6} = 120°$ and each exterior angle measures $\frac{360°}{6} = 60°$.

ii) A regular octagon has 8 congruent sides, so the measure of each interior angles is $\frac{(8-2)\times180°}{8} = 135°$.

2. For both cases 180°.

5.4. Properties of Regular Polygons

Periods allotted: 5 periods

Competencies

At the end of this subunit, students will be able to:

- find the line of symmetries of a regular polygon
- calculate the perimeters of regular polygons.
- calculate the areas of regular polygons.

Introduction

In this sub-unit students are required to find lines of symmetry of regular polygons, calculate the perimeter and area of a regular polygon.

To begin a lesson, students will recall what they have learned about symmetry of different figures by using activity and then give the formal definition of symmetry and line of symmetry.

On the next subsequent lessons, students should be assisted to explain a circumscribed polygon about a circle and inscribed polygon in a circle. Review definitions of central, interior and exterior angles. Ask students to define.

Central Angle: the angle formed by two adjacent vertices and the center of the polygon.

Interior Angle: the angle inside formed by two adjacent sides of a polygon.

Exterior Angle: the supplement of an interior angle of a polygon.

The main objective of this subunit is to derive the following formula inductively.

For n −sided regular polygon inscribed in a circle of radius r, length of side s, apothem a, perimeter P and area A are determined by

1. $s = 2r\sin\left(\frac{180°}{n}\right)$
2. $a = r\cos\left(\frac{180°}{n}\right)$
3. $P = 2nr\sin\left(\frac{180°}{n}\right)$
4. $A = \frac{1}{2}aP$

Answer for Activity 5.5

1.

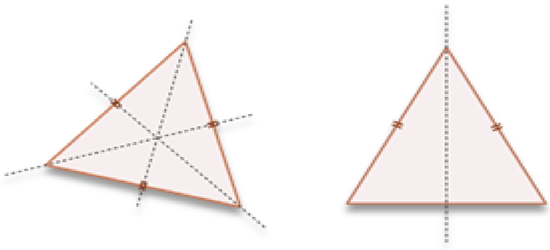


Figure 5.3

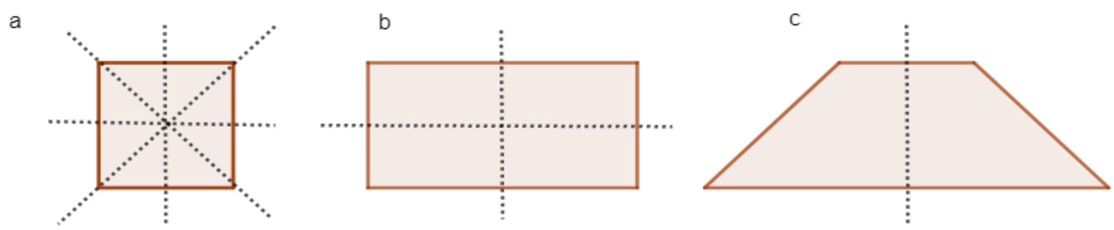
From figure 5.3 one can easily observe that

Equilateral triangle has 3 lines of symmetries and the isosceles triangle has 1 line of symmetry.

2. The number of lines of symmetry depends both on the number of sides of a polygon and the nature of the polygon. For example, in the above case both figures have the same number of sides but with different numbers of lines of symmetry.

Answer for Exercise 5.9

1.



Note that: For a trapezium, the line of symmetry is 1 for the case of isosceles trapezium as shown above and no line of symmetry if it is not isosceles.

2 A regular octagon has 8 lines of symmetry

3.

a. False (For example consider a rectangle, which is not regular but has line of symmetry)
b. True
c. True
d. False (There is no lines of symmetry passing through its sides)
e. True

Assessment

Use the graph paper and construct different polygons. Ask students to determine the number of lines of symmetry (they may cut out and check it by folding) , determine the measure of each interior and exterior angles using a protractor.

Answer for Activity 5.6

1. Construct the following plane figures

   a. A right angled triangle with sides 6 cm and 8 cm.
   b. A square of side 10 cm.

c. An equilateral triangle of side length 7 cm

F
i
g
u

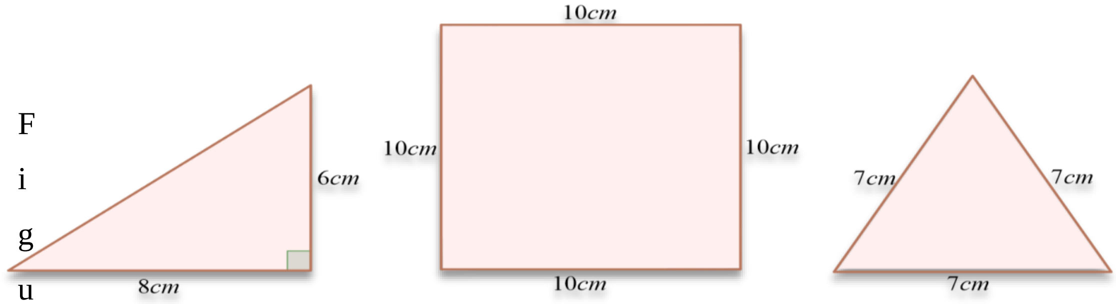


Figure 5.4

2 . Help students to practice to inscribe and circumscribed about the above plane figures.

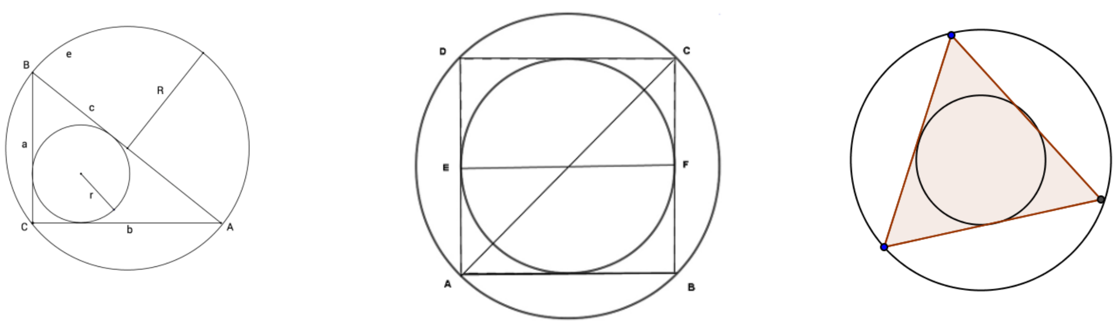


Figure 5.5

4 For regular polygons it is possible to construct such circles.

Answer for Exercise 5.10

1. **a** for $n = 10$, the measure of each central angle is $\frac{360°}{10} = 36°$.

b. similarly for $n = 15$, its central angle is $\frac{360°}{15} = 24°$.

2. You need to determine the number of sides of a regular polygon where its central angle is $12°$, so that $\frac{360°}{n} = 12°$. Hence, $n = 30$.

3. We check whether $\frac{360°}{central\ angle}$ a positive integer is or not.

a. $\frac{360°}{6°} = 60$, yes we can get a 60 sided regular polygon

b. No regular polygon with this central angle

c. No regular polygon with this central angle

Answer for Exercise 5.11

1. Given that a square is inscribed in a circle of radius 4 cm. We need to determine the side of the square (s), apothem (a), perimeter (p) and area of a square (A) .

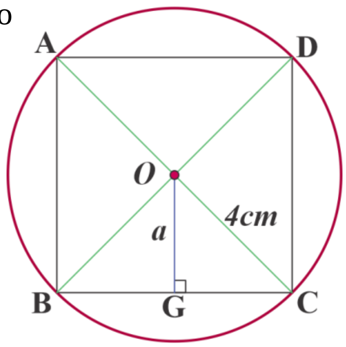


Figure 5.6

Consider triangle OGC, $m(\angle OCG)$=45°.

Hence, $\sin(\angle OCG) = \frac{a}{4}$. $\Rightarrow a = 2\sqrt{2}$ cm.

$\cos(\angle OCG) = \frac{GC}{OC} = \frac{GC}{4}$. $\Rightarrow GC = 2\sqrt{2}$ cm.

Therefore, $s = BC = 2 \times 2\sqrt{2} = 4\sqrt{2}$ cm. The perimeter of a square $p = 4s = 4 \times 4\sqrt{2} = 16\sqrt{2}$ cm.

The area of the square is $A = 4 \times \frac{1}{2} \times a \times s = 4 \times \frac{1}{2} \times 2\sqrt{2} \times 4\sqrt{2} = 32$ cm^2.

2. Given that an equilateral triangle is inscribed in a circle of radius. We need to determine the side of the square (s), apothem (a), perimeter(p) and area of an equilateral triangle (***A***) in terms of r .

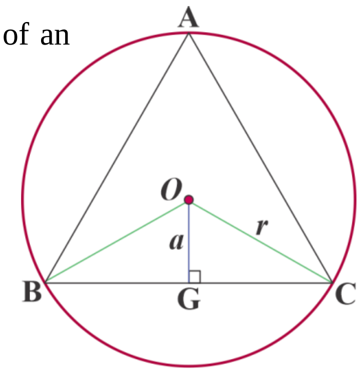


Figure 5.7

Consider triangle OGC, $m(\angle OCG)$=30°.

Hence, $\sin(\angle OCG) = \frac{a}{r}$. $\Rightarrow a = \frac{r}{2}$.

$\cos(\angle OCG) = \frac{GC}{OC} = \frac{GC}{r}$. $\Rightarrow GC = \frac{\sqrt{3}}{2}r$.

Therefore, $s = BC = 2 \times \frac{\sqrt{3}}{2}r = \sqrt{3}r$.

The perimeter of a triangle $= 3s = 3 \times \sqrt{3}r = 3\sqrt{3}r$.

The area of the triangle is $A = 3 \times \frac{1}{2} \times a \times s = 3 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \times 3\sqrt{3}r = \frac{27}{4}r$.

Answer for Exercise 5.12

1. Given that $n = 10$, and $r = 8$ cm we need to find

i) apothem $a = r\cos\left(\frac{180°}{n}\right) = 8 \text{ cm} \times \cos\left(\frac{180°}{10}\right) \approx 7.608$ cm

ii) perimeter $P = 2nr\sin\left(\frac{180°}{n}\right) = 2 \times 10 \times 8 \text{ cm} \times \sin\left(\frac{180°}{10}\right) \approx 49.443$ cm

iii) area $A = \frac{1}{2}aP \approx 188.081$ cm^2.

2. Since the central angle is $60°$, we can find the number of sides of the regular polygon. That is $\frac{360°}{n} = 60° \Rightarrow n = 6$, it is a regular hexagon. For regular hexagons, the side length of the polygon and the radius of the circle are the same. So that $r = s = 9$ cm.
3. Yes it is correct. In the construction of apothem, the radius is the hypotenuse of a right-angled triangle and apothem is one of the perpendicular legs of this right triangle. Hence, the hypotenuse is always the largest side of the right triangle, so it can never be shorter than either of the other two sides.

Assessment

You can use the review exercise for assessment of the subunit.

Summary

You have to recall the basic concepts in the unit orally. Summarize the main points of the unit. Focus on the topics how to determine the measure of internal and external angle of convex polygon, properties of regular polygons, area and perimeter of regular polygons inscribed in a circle. To check whether they understood or not provide the review exercise. Arrange a tutorial class and observe their reflection on the review exercise.

Answer for Review Exercises

1. a. False (take rectangle)
   b. True
   c. False (take a circle which is symmetric but not a polygon)
   d. False (a right angle triangle can be inscribed in a circle with the hypotenuse as a diameter).
   e. True
   f. True

2 a. First let us find he supplementary angle to x. From the picture the supplementary angles of the given exterior angles $64°, 125°, and\ 150°$ are $116°, 55°\ and\ 30°$, respectively.

Therefore,

$$x = 360° - (116° + 55° + 30°) = 21°$$

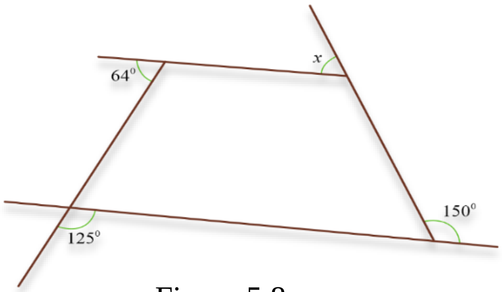


Figure 5.8

b. It is a six sided polygon so that the sum of interior angles of this polygon is $720°$.

Then, $20x = 720° \Rightarrow x = 36°$.

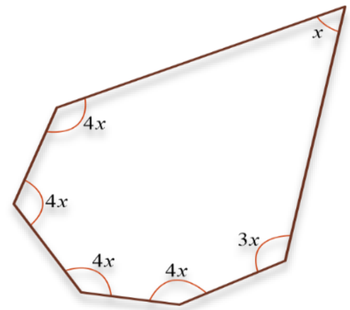


Figure 5.9

C. First let us find p and q and then r.

$p = 180° - 81° = 99°$.

$q = 180° - 68° = 112°$.

Since the polygon is a pentagon, the sum of the measure of interior angles of the polygon is $54°$.

Hence, $r = 540° - (99° + 165° + 110° + 112°)$

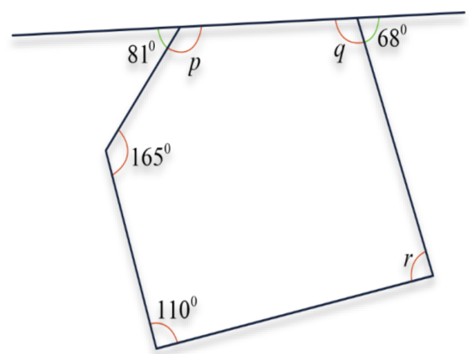


Figure 5.10

3. i) Suppose the measure of exterior angle be E and the measure of interior angle be I, given that $E = I$, since at each vertex of a polygon $E + I = 180° \Rightarrow 2E = 180°$ so that $I = 90°$. Now determine the polygon, that is $n = \frac{360°}{90°} = 4$. Hence, the regular polygon is a square.

ii) Again let the measure of exterior angle be E and the measure of interior angle be I, given that $E = 2I$, since at each vertex of a polygon $E + I = 180° \Rightarrow 3I = 180°$ so that $I = 60°$. Now determine the polygon, that is $n = \frac{360°}{120°} = 3$. Hence, the regular polygon is an equilateral triangle.

4. This is another way of asking to can we get a positive integer n which satisfy $(n - 2) \times 180° = 26 \times 90°$? Solving for n, we got $n = 15$. So the answer is yes, and it is possible.

5. The sum of exterior angles of any polygon is $360°$. $80° + 70n° = 360°$ (since one exterior angle is $80°$ and the rest are $70°$each). Solving for n we obtain $n = 4$. This is to mean, the polygon is 5 sided with four of its sides are congruent one with different sizes.

6. Let the interior angle of a pentagon be $2x, 3x, 5x, 9x \; and \; 11x$ for a five the corresponding ratios of interior angles $2: 3: 5: 9: 11$. Since it is a pentagon the sum of the measure of interior angles of a polygon is $540°$. Therefore, $2x + 3x + 5x + 9x + 11x = 540°$. Solving for x we obtain $x = 18°$. Hence, the interior angles of a polygon are $36°, 54°, 90°, 162°\ and\ 198°$. The given polygon is concave since one of the interior angle is greater than$180°$.

7. For a regular hexagon, the length of side (s) and radius (r) of a circle circumscribing it is the same. That is $= s$. So that let us use the formula $A = \frac{1}{2}ap = \frac{1}{2}rcos\left(\frac{180^{\circ}}{n}\right).2nrsin\left(\frac{180^{\circ}}{n}\right)$, where $n = 6, r = 10$ unit.

$$= \frac{1}{2} \times 10 \text{ unit} \times cos\left(\frac{180^{\circ}}{6}\right).2 \times 6 \times 10 \text{ unit} \times sin\left(\frac{180^{\circ}}{6}\right) = 150\sqrt{3} \text{ unit}^2.$$

Hence, the answer is 'C'.

8. The formula to determine perimeter (p) of an $n -$ sided regular polygon is $p = nr$ .The unknown term is r. To determine r , we use the area formula $A = \frac{1}{2}ap = \frac{1}{2}rcos\left(\frac{180^{\circ}}{n}\right).2nrsin\left(\frac{180^{\circ}}{n}\right)$. It was given that $n = 6$ and $A = 96\sqrt{3}\text{ cm}^2$. We need now to determine r. $96\sqrt{3}\text{ cm}^2 = \frac{1}{2}ap = \frac{1}{2} \times rcos\left(\frac{180^{\circ}}{6}\right).2 \times 6 \times r \times sin\left(\frac{180^{\circ}}{6}\right)$ which results $r^2 = 64\text{ cm}^2$. Since r represent radius we took the positive result. So that $r = 8$ cm. Hence, using $p = nr = 6 \times 8\text{ cm} = 48$ cm. (you can also use $p = 2nr\sin\left(\frac{180^{\circ}}{n}\right)$).

Answer is 'A'

9. **A.** Number of non-crossing diagonals from 1 vertex of $n -$ sided regular polygon is $n - 3$. Hence a regular octagon has $8 - 3 = 5$ diagonals.

B. The sum of the measures of all interior angles is $(n - 2) \times 180^{\circ} = 1080^{\circ}$.

C. The sum of the measures of all its central angles is 360°.

D. The measure of each exterior angle is $\frac{360^{\circ}}{n} = \frac{360^{\circ}}{8} = 45^{\circ}$.

So the correct answer is 'B'.

10. Answer is 'D' since the sum of the measure exterior angles is 360°.

11. The sum of the measure of exterior angles of a regular polygon is 360°. Hence, the measure of each exterior angle is $\frac{360^{\circ}}{n}$ where n is the no of sides of a polygon. Therefore $40^{\circ} = \frac{360^{\circ}}{n}$ results $n = 9$. So that the answer is 'B'.

12. The perimeter of a regular n −sided polygon inscribed in a circle of radius r is obtained by $p = 2nr\sin\left(\frac{180^{\circ}}{n}\right)$. It is given that $n = 5$ and $r = 10$ cm. Therefore,

$p = 2nr\sin\left(\frac{180^{\circ}}{n}\right) = 2 \times 5 \times 10 \text{ cm} \times \sin\left(\frac{180^{\circ}}{5}\right) = 100 \times \sin 36^{\circ} = 59$ cm. So that the answer is 'D'.

13. We need to check whether we get a positive integer n which satisfy $\frac{360°}{n} = 50°$. Here, we get $n = 7.2$ which could not be number of sides of a polygon. Therefore, there is no regular polygon with this exterior angle.

14. Given that the diameter of the square is $5\sqrt{2}$ cm. Using Pythagoras theorem, we can determine the length of side of the square as $2s^2$=50, where s stands for the length of side of the square. Solving for s we get $s = 5cm$.So that the perimeter of the square is $P = ns$, that is $p = 4(5 \text{ cm}) = 20$ cm.

15. At each vertex, the sum of interior and exterior angle is $180°$. Since it is regular polygon, let θ the exterior angle (it is the same for all vertex). Hence at any vertex $165° + \theta° = 180°$ results $\theta = 15°$. By the property of regular polygon, the measure of each exterior angle is $\frac{360°}{n}$ where n the no of sides of a polygon is. Hence, $\frac{360°}{n} = 15°$ results $n = 25$.

16. The two pentagons are regular so that, the measure of each interior angle is $\frac{(n-2)180°}{n} = \frac{3\times180°}{5} = 108°$. Also an equilateral triangle is ΔDEF and each interior angle measure $60°$. At vertex E,

$m(\angle AEI) + m(\angle IEF) + m(\angle DEF) + m(\angle DEA) = 360°$

$\Rightarrow m(\angle AEI) + 108° + 60° + 108° = 360°$

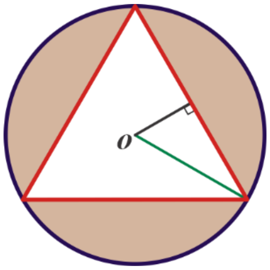
$\Rightarrow m(\angle AEI) = 360° - 276° = 84°$.

17. Since the polygon is regular, each side has equal length, the perimeter $p = ns$ where n is number of side of a polygon and s be the length of the side. Given that $p = 143$ unit and$s = 11\ unit$. So that $n = \frac{143unit}{11\ unit} = 13$. Hence the polygon is 13-gon.

18. Consider figure 5.11. We need area of the shaded region.
Given that the length of radius be 8 cm. Using the right angle triangle and applying Pythagoras theorem, the apothem is 4 cm.

And the length of side of a n equilateral triangle is $= 8\sqrt{3}$ cm .

Hence, area of the shaded region=area of the circle-area of an equilateral triangle



$$\Rightarrow A_{Shaded\ region} = \pi r^2 - \frac{1}{2}pa$$

$$\Rightarrow A_{shaded\ region} = (3.14)(64\ cm^2) - \frac{1}{2}(3 \times 8\sqrt{3})(4cm) = 117.92\ cm^2.$$

Figure 5.11

19. For each interior angle we use a formula $\frac{(n-2)\times 180°}{n}$ and for each exterior angle we use $\frac{360°}{n}$ where n is the number of sides of a polygon.

The maximum exterior angle is possible when n is the least possible number. The least possible number of side of a polygon is $n = 3$. Using this in the interior angle formula we find each interior angle of an equilateral triangle is 60°, so that the maximum possible exterior angle is 120°. No other regular polygon has such property.

20. Suppose one of the sides of the octagon be AB as shown in figure 5.12

The central angle is $\angle AOB$ where θ is half of it. For regular octagon the measure of a central angle is $\frac{360°}{8} = 45°$, so that $\theta = 22.5°$. The length of side of the stop sign is $AB = 40cm$ so that $AD = 20cm$. Now the apothem (a) will be determined as

$\tan 22.5° = \frac{AD}{a}$ ⇒which implies $a \approx 48.2843$ cm.

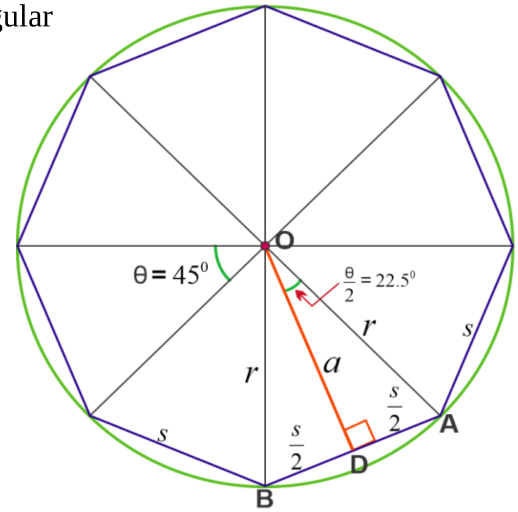


Figure 5.12

Hence, the area of the stop sign will be determined by using the formula

$A = \frac{1}{2}aP$ where p is the perimeter and a is apothem.

$p = ns = 8 \times 40$ cm $= 320$ cm

Therefore, $A = \frac{1}{2}(48.2843\text{ cm})(320\text{ cm}) \approx 7{,}725.488\text{ cm}^2$.

So that the area of the stop sign is 7,725 cm^2 to the nearest integer.

Unit 6

CONGRUENCY AND SIMILARITY (15 periods)

Introduction

Within the field of mathematics, and in particular geometry, *congruency* and *similarity* are related terms. *Congruence* basically means that two figures or objects are of the same shape and size. Even though congruent objects are identical, their orientations with respect to one another and their physical coordinates in a plane or three-dimensional space, may often be different.

Similarity means that two figures or objects are of the same shape, though usually not of the same size. For example, two circles will always be similar because by definition they have the same shape. If the circles have radii of different lengths, however, they will not be congruent.

After completing this unit, students will be able to:

- identify similar figures;
- distinguish between congruent and similar plane figures;
- state and use the criteria for similarity of triangles viz. AAA, SSS and SAS;
- prove the Pythagoras Theorem;
- prove that if a line is drawn parallel to one side of a triangle then the other two sides are divided in the same ratio;
- apply these results in verifying experimentally (or proving logically) problems based on similar triangles.

6.1 Revision on Congruency of Triangles

Periods allotted: 1 Period

Competencies

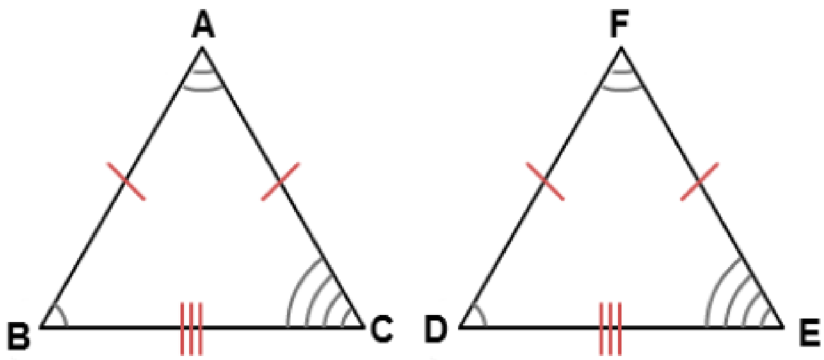
- Use the postulates and theorems on congruent triangle in solving related problems.

Key words: Congruency, Triangles

In case of congruent triangles-

- All the sides of one triangle must be equal to the corresponding sides of another triangle.

- All the angles of one triangle must be equal to the corresponding angles of another triangle.
- All the vertices of one triangle must be corresponding to the vertices of another triangle.



In the above triangles,

If, ΔABC ≡ ΔFDE then

- Corresponding vertices are – ∠A ↔ ∠F, ∠B ↔ ∠D and ∠C ↔ ∠E
- Corresponding angles are - ∠A ↔ ∠F, ∠B ↔ ∠D and ∠C ↔ ∠E
- Corresponding sides are – AB ↔ FD, BC ↔ DE and AC ↔ FE

Remark: It is the order of the letters in the names of congruent triangles which tells the corresponding relationships between two triangles. If we change it from ΔABC ≡ ΔFDE to ΔBCA ≡ ΔFDE, then it is not necessary that the two triangles are congruent as it is important that all the corresponding sides, angles and vertices are same.

Answers to Activity 6.1

a. If two triangles are congruent, then they have the same shape and size.

b. Their corresponding angles and sides are congruent.

c. ii, $\angle A \equiv \angle D$ because it leads to an established congruence theorem called ASA.

Answers to Exercise 6.1

a) True **b)** False **c)** Triangles A and B, B and C, A and C are congruent

d) ∠D = 45°, ∠B= ∠E = $180 - (85 + 45) = 50$

Answers to Exercise 6.2

b & e (by SAS), d & f (by SSS)

Ask students to relate AAS with one of SSS, SAS or ASA congruency theorem. Consider the triangles below.

Answers to Exercise 6.3

No two triangles are congruent by AAS

Answers to Exercise 6.4

a) 1 & 5, 3 & 6, 2 & 4

b)

i. The three sides are congruent to the corresponding three sides. From SSS congruency, ABC is congruent to PQR

$\Delta ABC \equiv \Delta PQR$

ii. Two angles and included side are equal ,so from ASA congruence, ABC is congruent to PQR

$\Delta ABC \equiv \Delta PQR$

iii. Two sides are equal but the angle is not the included ,so ABC is not congruent to PQR

iv. Two sides are equal and included angle is equal so from SAS congruence, ABC is congruent to PQR

$\Delta ABC \equiv \Delta PQR$

v. Two angles and other side are equal ,so from AAS congruence, ABC is congruent to PQR

$\Delta ABC \equiv \Delta PQR$

vi. Just two sides are congruent, So ABC is not congruent to PQR

vii. Just two(of course three) angles are equal, there is no information about side, and So ABC is not congruent to PQR

viii. Two angles are congruent and included side is equal. So from ASA congruence, ABC is congruent to PRQ

$\Delta ABC \equiv \Delta PQR$

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercises and problems on identifying congruency of triangles and/or tests/quizzes. Consider two triangles drawn in a grid. Ask students whether they are congruent or not? Ask them to explain the reason.

6.2 Definition of Similar Figures

Periods allotted 2

Competencies

- define similar plane figures.

Key words: Similar, figures

Answers to Activity 6.2

1. Having the same number of sides does not necessarily mean that the triangles are similar.
2. If two plane figures have the same area they may not be similar. For instance, a triangle and a rectangle could have equal area; however, they are not similar. On the other hand, if two triangles are congruent, then they have equal area and also are similar.

Answers to Exercise 6.5

False. Consider an equilateral and a square. They have different shapes; hence they cannot be similar.

Answers to Exercise 6.6

a) $x = 12 \; and \; y = 18$

b) $72 \; cms$

c) The common ratio of their corresponding sides is $\frac{4}{6} = \frac{2}{3}$. $BC = 3cm$

Answers to Exercise 6.7

a. True

b. False

c. True

d. False

e. True

f. True

g. True

h. False

i. True

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercises and problems on definition of similar figures and/or tests/quizzes. Consider two hexagons. Ask students to determine whether the hexagons are similar or not and generalize for all regular n-gons. Ask them to explain why?

6.3. Theorems on Similar Plane Figures

Periods allotted 2

Competencies

- apply the SSS, SAS and AA similarity theorems to prove similarity of triangles

Key words: Theorem, Similar, plane figures.

If **GeoGebra** (You can get it in some universities or you can download in the internet) is available at your school, please show the students how similarity of figures (or /and congruency) can be easily visualized.

Answers to Activity 6.3

The quadrilaterals are not only similar but also congruent. Why? Because the corresponding angles and the corresponding sides are not only proportional but also congruent. The triangles are similar. Why? Because the corresponding angles are congruent and the corresponding sides are proportional.

Answers to Exercise 6.8

Show for students by checking the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

(a) The two triangles are similar because the corresponding angles are congruent and the ratios of the corresponding sides are congruent.

(b) The pairs in (1) are similar by AA similarity theorem. The pairs in (2) are not similar

Answers to Exercise 6.9

a) Since $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{2}{3}$, by SSS, the triangles are similar.

b) (1) The pairs of the triangles are similar by SSS. (2) are not similar. (3) similar by SSS.

Answers to Exercise 6.10a

a) By SAS similarity theorem, the two triangles are similar.

b) There are no two similar triangles.

c) $\angle D = \angle I = 116^0$ and Since the polygons are similar, $\frac{JF}{EA} = \frac{GH}{BC}$. $\frac{JF}{4.4} = \frac{4.4}{2.2}$. $JF = 8.8$. Similarly, IJ = 6.4

Answers to Exercise 6.10b

1. **a)** $\Delta ABE \sim \Delta ACD$ by AA similarity theorem

b) $\frac{BC}{EF} = \frac{AC}{DF} = \frac{AB}{DE}$. Therefore, $\frac{BC}{12} = \frac{20}{15}$. Hence, $BC = 16$

2. When two triangles are congruent, their corresponding angles and corresponding sides are congruent. Two triangles are similar, if they have the same shape. When two triangles are congruent, then they are similar.

3. **a)** $\widehat{B} \equiv \widehat{H}$, $\widehat{I} \equiv \widehat{A}$, $\widehat{G} \equiv \widehat{T}$ & $\frac{BI}{HA} = \frac{IG}{AT} = \frac{BG}{HT}$

b) The scale factor is $\frac{3}{5}$ or $\frac{5}{3}$

c) $HT = 35$

d) $IG = 27$

4. **a.** From the figure, $C\hat{A}B \equiv D\hat{E}F$ & $C\hat{B}A \equiv DF\hat{F}E$. So $\Delta ABC \sim \Delta EFD$, that is the triangles are similar by AA similarity theorem.

   b. x = 8.

5. a) $x = 5\sqrt{3}$ *and* $y = 5\sqrt{6}$. b) $x = \frac{20}{9}$

   c) $x = \frac{14}{3}$. d) $x = 16,\ y = 8\sqrt{5}$ *and* $z = 8\sqrt{5}$

6. **a)** $\Delta ABE \sim \Delta CDE$ by AA similarity theorem

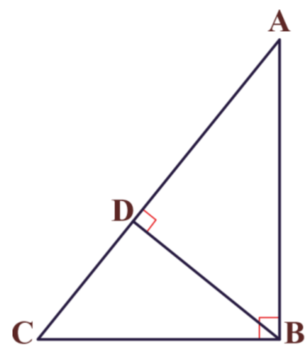
   b) ΔAED & ΔBEC

   c) $\frac{AB}{CD} = \frac{BE}{DE} = \frac{AE}{CE}$. Hence, $\frac{10}{22} = \frac{7}{CE}$. Therefore, $CE = 15.4$. This implies that $AC = AE + CE$.

   So, $AC = 7 + 15.4 = 22.4$

7. Consider ΔABC be a right angel triangle.

   To prove: $AC^2 = AB^2 + BC^2$



Draw BD perpendicular to AC.

$\Delta ADB \sim \Delta ABC$ by AA similarity theorem.

Hence, $\frac{AB}{AC} = \frac{AD}{AB}$. Therefore, $AB^2 = (AC)(AD)$…………..(i)

and $\Delta BDC \sim \Delta ABC$ by AA similarity theorem.

Hence, $\frac{BC}{AC} = \frac{DC}{BC}$. Therefore, $BC^2 = (AC)(DC)$…………….(ii)

Adding (i) and (ii), we have $AB^2 + BC^2 = (AC)(AD) + (AC)(DC)$

$$AB^2 + BC^2 = AC[AD + DC] = (AC)(AC)$$

Hence, The pythagoras theorem, $AC^2 = AB^2 + BC^2$

8. $\Delta ACB \sim \Delta ADC$ by AA similarity theorem.

   Hence, **i.** $\frac{AC}{AD} = \frac{BC}{DC} = \frac{AB}{AC}$. So, $\frac{b}{x} = \frac{a}{p} = \frac{c}{b}$. Therefore, $pc = ab$

ii. $p = \frac{ab}{c}$ implies that $p^2 = \frac{a^2b^2}{c^2} = \frac{a^2b^2}{a^2+b^2}$. Taking the reciprocal, we get

$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2}$. Hence, $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

9. Drop a perpendicular from E to their common base and produce other two smaller right triangles. Use similarity of the original right triangles with the newly produced right triangles. Finally, you will get the perpendicular distance from the point of intersection E to their common base is $h = \frac{15}{8}$. Observe that the answer is independent of the length of the common base. Is there any other method to solve this problem? Use **the cross-ladder theorem** i.e.$\frac{1}{h} = \frac{1}{AB} + \frac{1}{CD}$, where AB and CD are the parallel legs of the two right triangle. $\frac{1}{h} = \frac{1}{5} + \frac{1}{3}$. Solving for h we have $h = \frac{15}{8}$. What can you conclude? The answer is equal to the product of the two parallel legs of the right triangles divided by their sum.

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercises and problems on similarity theorems and/or tests/quizzes. Consider any two triangles with two pairs of their respective sides are proportional and one pair is not proportional. Ask students whether the triangles are similar or not. Consider two triangles with two pairs of proportional sides and non-included angles. Ask students whether the triangles are similar or not and ask them to explain why?

6.4. Ratio of perimeters of similar plane figures

Periods allotted 2

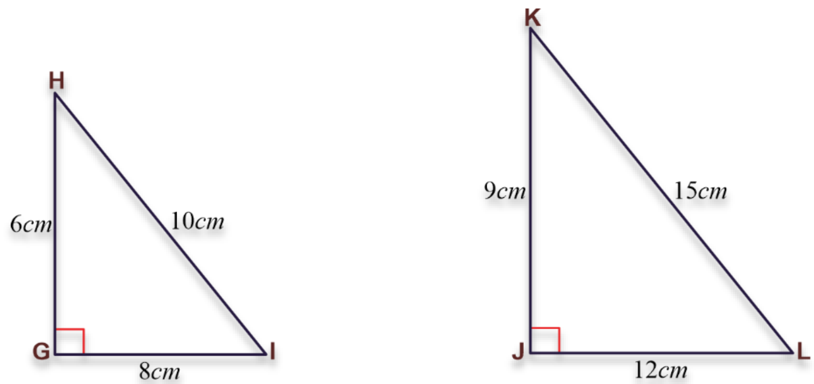
Competencies

- discover the relationship between the perimeters of similar plane figures and use this relationship to solve related problems.

  Key words: Ratios, Perimeters, similar plane figures

When two plane figures are similar, the reduced ratio of any two corresponding sides is called the **scale factor** of the similar figures. Give enough number of similar plane figures including triangles and let the students conclude the ratios of the perimeters of the similar plane figures is the same as the ratio of the corresponding sides.

For example, consider the following:



The triangles above are congruent. ΔGHI~ΔJKL by AA similarity theorem.

$\frac{GH}{JK} = \frac{HI}{KL} = \frac{GI}{JL}$. $\frac{6}{9} = \frac{10}{15} = \frac{8}{12} = \frac{2}{3}$. Therefore, the ratio of their perimeter is

$$\frac{6+8+10}{9+12+15} = \frac{24}{36} = \frac{2}{3}$$

Answers to Activity 6.4

1. **Ratio** is a comparison of two or more quantities (magnitudes) of the same kind and in the same unit
2. A **scale factor** is simply a number that multiplies the dimensions of a shape. This can make a shape larger or smaller. Larger shapes will have a scale factor greater than one. On the other hand, smaller shapes will have a scale factor less than one. So, if the scale factor is three, then the dimensions of the new shape will be three times larger than that of the original. If the scale factor is half, then the dimensions of the new shape will be two times smaller than that of the original.
3. Use **pattern blocks**, as indicated in activity 6.4 in the text, to make a figure whose dimensions are 2, 3, and 4 times greater than those of the original figure and generalize it. Form different groups to do this activity.
4. As can be seen from the pattern, as the sides are doubled the perimeters are also doubled and so on. Therefore, the ratio of their perimeter **is equal to** the ratio of their corresponding sides. However, from the pattern given, one can conclude that the ratio of their area is equal to the ratio of the square of their corresponding sides.

Answers to Exercise 6.11

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. $The\ ratio\ of\ the\ smaller\ triangle\ to\ the\ larger = \frac{2}{6} = \frac{1}{3}$
2. $3:5\ or\ 5:3$
3. $2:3\ or\ 3:2$
4. Doubling the side lengths of a rectangle doubles its perimeter.
5. Quadrupling the side lengths of a rectangle quadruples its perimeter.
6. $\frac{11}{6}, \frac{4}{7}, \frac{5}{8}$ and $\frac{14}{9}$, respectively.

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercises and problems on ratios of perimeters of similar plane figures and/or tests/quizzes. Consider two pairs of regular pentagons with each side length equal to 3 units and 5 units. Ask the students the following questions: Are the two regular pentagons similar? Find the ratio of the perimeters of the pentagons. Ask them to generalize the problem.

6.5. Ratio of areas of similar plane figures

Periods allotted 3

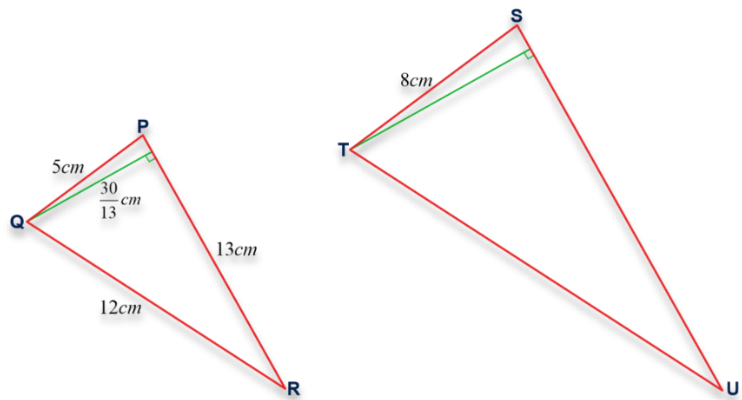
Competencies

- discover the relationship between the areas of similar plane figures and use this' relationship to solve related problems.

  Key words: Ratios, Areas, similar plane figures.

How do changes in dimensions of similar geometric figures affect the areas of the figures? Use pattern blocks, as indicated in activity 6.4, to make a figure whose dimensions are 2, 3, and 4 times greater than those of the original figure and then generalize it. Form different groups to do this activity.

Consider the similar triangles below:



In the figures above, $\Delta PQR \sim \Delta STU$ by AA similarity theorem.

$\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta STU} = \left(\frac{5}{8}\right)^2 = \frac{25}{64}$. Area of $\Delta PQR = 30$

Area of $\Delta STU = \frac{(64)(30)}{25} = 76\frac{4}{5}$

Answers to Activity 6.5

1. From the pattern given, one can conclude that the ratio of their area is equal to the ratio of the square of their corresponding sides.
2. Ratio of thier sides $= \frac{3.7}{7.4} = \frac{1}{2}$. Ratio of thier areas $= \frac{9.2}{36.8} = \frac{1}{4} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ (If you drag any orange dot (vertex) at P, Q, R, you will get similar triangles. Would you please use GEOGEBRA)

Answers to Exercise 6.12a

The ratio of the areas of the rhombi is $\frac{6^2}{10^2} = \left(\frac{6}{10}\right)^2 = \frac{9}{25}$

Answers to Exercise 6.12b

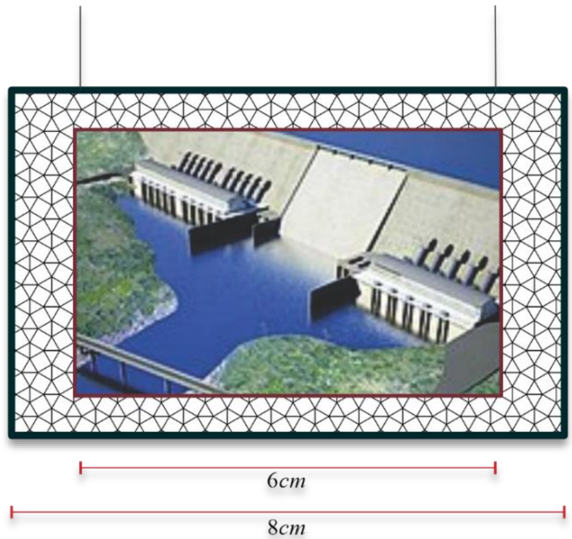
Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. $\frac{6}{3} = 2 \ or \ \frac{3}{6} = \frac{1}{3}$
2. The scale factor is $8:7$ and the ratio of their perimeters is$8:7$
3. The ratio of their circumferences is $2:3 \ or \ 3:2$ and the ratio of their areas is $4:9 \ or \ 9:4$
4. The scale factor is $5:6$ and the ratio of their perimeters is $5:6$.

5. The area of the second triangle is 108 square unit.
6. The length of the corresponding side of the smaller triangle is $24cm$.
7. Doubling the side lengths of a rectangle quadruples its area.
8. Quadrupling the side lengths of a rectangle multiplies its area by Quadruple Square.
9. **a.** Find the ratio of the area of the page to the area of the picture.

$$\frac{\text{Area of the page}}{\text{Area of the picture}} = \left(\frac{\text{length of the page}}{\text{length of the picture}}\right)^2 = \left(\frac{8}{6}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Therefore, the area of the page is $\frac{16}{9}$ times greater than the area of the picture.



b. Multiply the area of the picture by $\frac{16}{9}$.

$45\left(\frac{16}{9}\right) = 80$

Hence, the area of the page is 80 square units.

The area of the page is $\frac{16}{9}$ times the area of the picture. Why greater? As page/ picture, which is the scale factor (16/9), is greater than 1; the area of the page is greater than the area of the picture.

10. A triangle with an area of 10 square meters has a base of 4 meters. A similar triangle has an area of 90 square meters. What is the height of the larger triangle?

$$\frac{\text{Area of the smaller triangle}}{\text{Area of the larger triangle}} = \left(\frac{\text{base of the smaller triangle, } b_1}{\text{base of the larger triangle, } b_2}\right)^2 = \frac{10}{90} = \left(\frac{10}{b_2}\right)^2$$

Hence, the larger triangle has base, $b_2 = 30$ meters. Hence, area of the larger triangle which is $90 = \frac{1}{2}(\text{base})(\text{height})$. Hence, $90 = \frac{1}{2}(30)(\text{height})$. Therefore, the height of the larger triangle is equal to 6 meters.

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercises and problems on ratios of areas of similar plane figures. and/or tests/quizzes. Consider two pairs of hexagons with each side length equal to 3, 6, 2, 4, 5, 2 and 9, 18, 6, 12, 15, 6. Ask the students the following questions: Are the polygons similar? Find the ratio of the areas of the hexagons. Ask them to generalize the problem and the solution.

6.6. Construction of similar figures

Periods allotted 3

Competency

- able to construct similar figures.

Key words: Construction, Similar figures.

Answers to activity 6.6

1. A **scale factor** is simply a number that makes a shape larger or smaller.
2. **Assist** students in drawing parallel line segments.
3. If the scale factor is greater than1, the figure to be constructed will be larger. If the scale factor is less than1, the figure to be constructed will be smaller. If equal to 1, the newly constructed figure is congruent to the original.

You guide your students by following step by step guidance. There are three cases.

Case (i): Scale factor 2

This is the common case that they learned in primary level.

The other two cases are explained below along with the required steps.

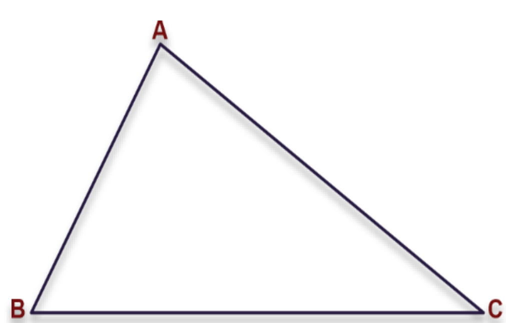
Case (ii)

Construct a triangle similar to the given triangle ABC for example, with **scale factor** $\frac{5}{3}$.

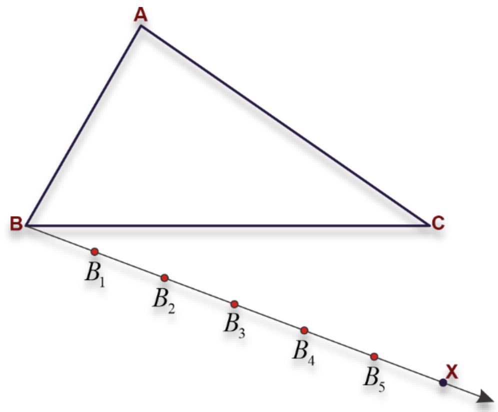
Here, scale factor $\frac{5}{3}$ means, the new triangle will have side lengths $\frac{5}{3}$ times the corresponding side lengths.

Steps of construction:

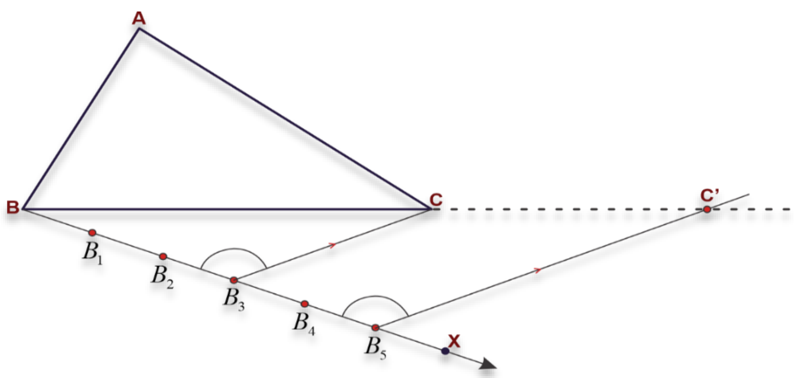
Step 1: Construct the triangle ABC as given below:



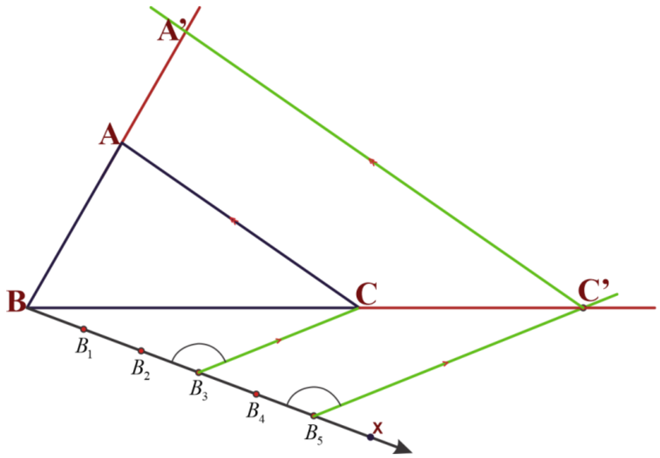
Step 2: Draw a ray BX making an acute, acute with the base BC and mark 5 points B_1, B_2, B_3, B_4, B_5 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.



Step 3: Join B_3C and draw a line B_5C' such that B_3C is parallel to B_5C' , where C' lie on the produced BC.



Step 4: Now draw another line parallel to AC at C' such that it meets the produced BA at A'.

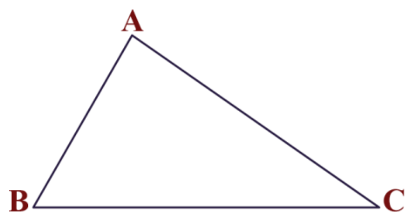


Hence, $\Delta A'B'C'$ is the required triangle similar to the ΔABC.

Case (iii)

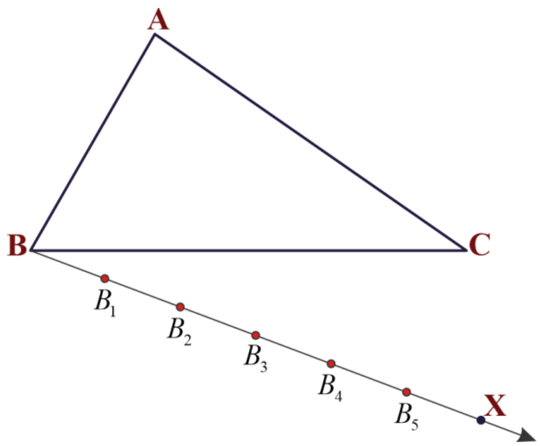
Construct a triangle which is similar to ΔABC with **scale factor $\frac{3}{5}$.**

Here, scale factor $\frac{3}{5}$ means, the new triangle will have side lengths $\frac{3}{5}$ times the corresponding side lengths of the given triangle. Consider the triangle ABC below:

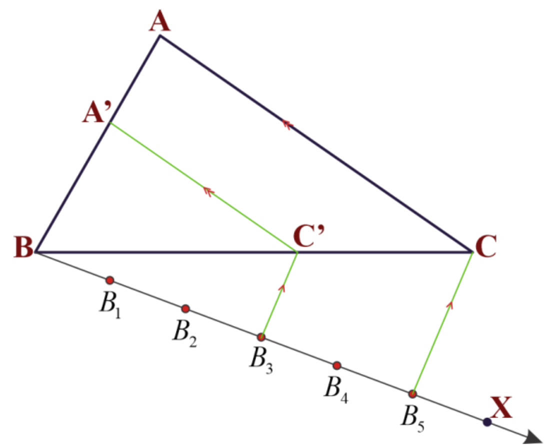


Steps of Construction:

1. Draw a ray BX which makes an acute angle with BC on the opposite side of vertex A.
2. Locate 5 points on the ray BX and mark them as $B_1,\ B_2,\ B_3,\ B_4, B_5$ on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
3. Join B_5C



4. Draw a line parallel to B_5C through B_3 (since 3 is the smallest among 3 and 5) and mark C' where it intersects with BC.
5. Draw a line through the point C' parallel to AC and mark A' where it intersects AB.
6. $A'BC'$ A is the required triangle.



Answers to Exercise 6.13

Apply those discussed above to construct similar triangles with the given scale factor.

ASSESSMENT

You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercises and problems on construction of similar plane figures and/or tests/quizzes. Consider a triangle with sides 3, 4 and 5. Construct a triangle similar to the given triangle.

6.7 Applications of similarity

Periods allotted 2

Competencies

- solve real life problems using the concepts of similarity and congruency.
- discover the relationship between the area of similar plane figure and use this relationship to solve related problems.

Key words: Application, Similarity.

Merema is standing outside next to a flagpole. The sun casts a 4 m. shadow of Merema and a 7 m shadow of the flagpole. If Merema is 1.5m tall, how tall is the flagpole? Without a ladder and measuring stick, you may think that solving this problem is impossible. However, finding the solution may be easier than you think. For this problem, and others like it, solving becomes a matter of **similar triangles**.

Answers to Activity 6.7

Without actually measuring the height of the triangle, using similarity of triangles, the length of the top of the ladder from to the ground is $\frac{10}{3}$

Answers to exercise 6.14

Find two similar triangles $\Delta AEC \sim \Delta BDC$. Then, $\frac{1.8}{BD} = \frac{2.4}{2}$, which implies that the height of Abdi's brother is $BD = 1.5m$

Answers to exercise 6.15

1. $3m$
2. $4.05m$
3. $150cm$
4. $10m$
5. $5m$
6. **i.** $AC^2 + BC^2 = AB^2 \ldots \ldots \ldots \ldots$ Pythagoras theorem

   ii. $AC^2 + QC^2 = AQ^2 \ldots \ldots \ldots \ldots$ Pythagoras theorem

   iii. $PC^2 + QC^2 = PQ^2 \ldots \ldots \ldots \ldots$ Pythagoras theorem

   iV. $PC^2 + BC^2 = BP^2 \ldots \ldots \ldots \ldots$ Pythagoras theorem

   Adding ii) and iv), we get $AQ^2 + BP^2 = AC^2 + QC^2 + PC^2 + BC^2$

   $$AQ^2 + BP^2 = (AC^2 + BC^2) + (QC^2 + PC^2)$$

   $$AQ^2 + BP^2 = AB^2 + PQ^2$$
7. $31.2\ cm$
8. 17m

ASSESSMENT

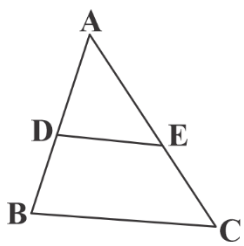
You can use any one of assessment techniques such as: class activities, group discussions, homework/assignments, exercises and problems on applications of similar plane figures.

and/or tests/quizzes. Consider a 13m ladder lying on a 12m vertical wall. Ask students to find the length from the foot of the ladder to the wall.

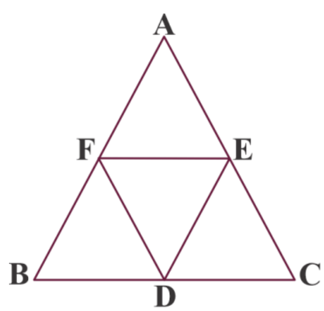
Solutions to Review Exercises on Unit 6

Show for students by checking some of the answers using Geogebra, Mathlab, mathematica or similar tools. Please appreciate students to use IT in mathematics.

1. $\Delta ABE \equiv \Delta CBD$ by ASA. Hence, $AE \equiv CD$
2. By RHS, $\Delta ADB \equiv \Delta ADC$. Hence, $BD \equiv CD$. Therefore, AD bisects BC
3. $\Delta ABP \equiv \Delta ABQ$ by RHS. Hence, $BP \equiv BQ$
4. Consider the triangle below. Line segment DE is Parallel to line segment BC(given). Hence, $\angle ADE \equiv \angle ABC$ and $\angle AED \equiv \angle ACB$. So, $\Delta ADE \sim \Delta ABC$



5. Consider the triangle below satisfying all the conditions



Since the segment joining the mid-points of the sides of a triangle is half of the third side. Therefore,

$$DE = \frac{1}{2}AB \text{ implies } DE = AF = BF \ldots \ldots (i)$$

$$EF = \frac{1}{2}BC \text{ implies } EF = BD = CD \ldots \ldots (ii)$$

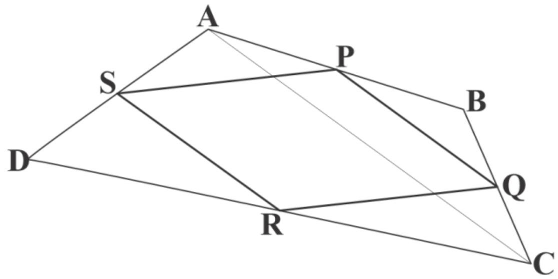
$$DF = \frac{1}{2}AC \text{ implies } DF = AE = EC \ldots \ldots (iii)$$

Hence, in ΔDEF and ΔAFE

$DE = AF$, $DF = AE$ and $EF = FE$, so, $\Delta DEF \equiv \Delta AFE$ by SSS

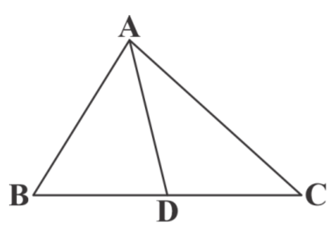
Similarly, $\Delta DEF \equiv \Delta FBD \equiv \Delta EDC$. Consequently, they are similar.

6. Consider the quadrilateral $ABCD$ and its mid-points $P, Q, R, and\ S$ as follows:



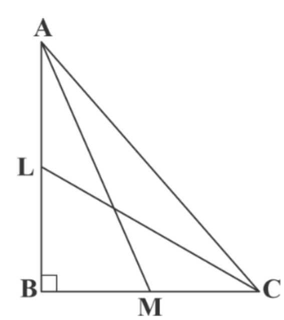
1. $AP = PB$……………… Given
2. $BQ = QC$……………… Given
3. $PQ//AC$……………1), 2)……Triangle mid segment theorem
4. $PQ = \frac{1}{2}AC$…………1), 2) Triangle mid segment theorem
5. $AS = SD$………………..Given
6. $CR = RD$………………..Given
7. $SR//AC$…………….5), 6), Triangle mid segment theorem
8. $SR = \frac{1}{2}AC$…………..…5), 6),Triangle mid segment theorem
9. $SR = PQ$ …….4), 8), Transitive property of equality
10. $SR//PQ$…3), 7) Two lines parallel to a third are parallel to each other.
11. $PQRS$ is a Parallelogram (Quadrilateral with two opposite sides that are parallel & equal)

7. Consider the triangle below:



Since $\angle BAC \equiv \angle ADC$ and $\angle C \equiv \angle C$, $\triangle ADC \sim \triangle BAC$ by AA similarity theorem.

Hence, we get $\frac{AD}{BA} = \frac{DC}{AC} = \frac{AC}{BC}$ implies $AC^2 = CB.CD$



8. Since L and M are the mid-points of AB and , we get $AL = LB$ and $BM = MC$.

Take $\triangle LBC$ and apply Pythagoras theorem. $(LB)^2 + (BC)^2 = (LC)^2$ implies

$\left(\frac{AB}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 = (LC)^2$

After simplification, we get $4LC^2 = AB^2 + 4BC^2$

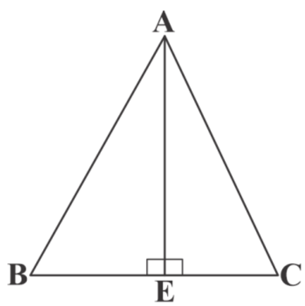
9. If two triangles are similar, then the ratio of their area is equal to the ratio of the square of the corresponding sides. Hence, $\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2 = \left(\frac{12}{18}\right)^2 = \frac{4}{9}$. So, the ratio is $4:9$

10. If two triangles are similar, then the ratio of their area is equal to the ratio of the square of the altitudes. Hence, $\frac{A_1}{A_2} = \left(\frac{2.5}{3.5}\right)^2 = \left(\frac{5}{7}\right)^2 = \frac{25}{49}$. So, the ratio is $25:49$.

11. The distance between the tops of the poles is 13 (use construction then Pythagoras's theorem). From the problem, you will get two similar triangles (By construction). Then, $\frac{17}{12} = \frac{x+12}{x}$, where x is the distance from the foot of the smaller pole to the ground after the extension of the line segment joining the two topes.

12. Let L be the length of the ladder. Then, $L^2 = 6^2 + 8^2 = 100$ implies $L = 10$

13. Consider the equilateral triangle below:



Use the Pythagoras theorem for $\triangle ABE$. $AB^2 = AE^2 + BE^2$. Let x be a side of the equilateral triangle. $AB = x$, $BE = \frac{x}{2}$. Then, $AE^2 = AB^2 - BE^2$.

$AE^2 = x^2 - \left(\frac{x}{2}\right)^2$ implies $AE^2 = 3\frac{x^2}{4}$.

Hence, $4AE^2 = 3x^2$. Therefore, three times the square of a side equals four times the square of a median for any equilateral triangle.

14. If two triangles are similar, then the ratio of their areas equals the square of the ratio of their corresponding base. Let b be the base of the smaller triangle. Therefore, $\frac{60}{50} = \frac{4}{b}$

Hence, the corresponding base of the smaller triangle is $b = \frac{10}{3}$.

15. *C*

16. *B*

17. *A*

Unit 7

VECTORS IN TWO DIMENSIONS (14 periods)

Introduction

In our day to day life, we come across many queries such as:-How old are you? What is the distance between city A and B? How should a football player hit the ball to give a pass to another player of his team? Here are the possible answers: It could be 40 years and for the first and 50 km for the second. In both cases, the reply is a quantity that involves only one value (magnitude) which is a real number. Such quantities are called scalars. However, an answer to the third query is a quantity (called force) which involves muscular strength (magnitude) and direction (in which another player is positioned). Such quantities are called vectors. In mathematics, physics and engineering, we frequently come across with both types of quantities, namely, scalar quantities such as length, mass, time, distance, speed, area, volume, temperature, work, money, voltage, density, resistance etc. and vector quantities like displacement, velocity, acceleration, force, weight, momentum, electric field intensity etc. So, this unit introduces the basics of vector and scalar quantities. After they identify the two quantities, they are expected to represent vectors of two dimensions and perform operations on them (addition, subtraction and scalar multiplication of vectors). Discussion on operation of vectors will be conducted graphically and in terms of component position vector. For such quantities, trigonometric ratio is important to describe the direction of the vector, so order your students to revise the unit four. In each of the problem try to relate the discussion with practical problem.

Unit Learning Outcomes

After completing the unit, students will be able to:

- **conceptualize vectors in the sense of direction and magnitude symbolically.**
- **perform operations on vectors**.

Suggested Teaching Aids for unit 7

Students have different experience about the measurement and different types of physical quantities. Ask students to list such physical quantities even from other science subject lessons. Most of the quantities in the list might be visible, you can consider them as a teaching material and they can imagine them without bringing the object or the material to the class room. For instance on the main road cars and people are moving in opposite direction, which might be taken as parallel vectors with opposite direction.

For this unit, the students need to bring rulers, compass, protractor, colored pen and other important instruments and use it whenever necessary. You need to use colored chalk (different colored markers if you are using white board) and use graphical paper.

If the school has a computer laboratory, use geogebra, matlab or any related mathematical software.

7.1 Vector and Scalar Quantities

Periods allotted: 2 periods

Competencies

At the end of this subunit, students will be able to:

- define scalar quantity
- define vector quantity
- differentiate vectors from scalar quantities

Introduction

The main concern of this sub unit is introducing vectors and scalars. Students have experience about the physical quantity even though they did not categorize them as vectors and scalars. So that after completion of this subunit, students are expected to differentiate vectors from scalars.

Teaching notes

To understand about the previous knowledge of students on about vector and scalar quantity, ask students a question like 'Can you give me any physical quantity you know from your environment or from other science classes?' Then list each of them on the blackboard, which might be part of activity 7.1. Then give time to think and reflect which of them need direction and which of them do not need direction. While you are asking the question of listing physical quantities, they may reflect different unit of measurement. Encourage them to express their idea freely.

Answers for Activity 7.1

a. There are different answers, some of them may be

   Mass, height, velocity, force, density, speed, temperature, energy, acceleration, etc

b. This activity will help students to categorise quantities to scalar and vector, so help students that the list of physical quantities have the same property, that is property of a material or a system which is quantified by measurement.

   They differ and could be categorized as those which has direction and do not have direction.

Answers for Exercise 7.1

1. D (from definition of vector quantity)
2. C
3. **i)** Scalar **ii)** Scalar **iii)** Vector **iv)** Scalar

   v) Scalar **vi)** Vector **vii)** Scalar **viii)** Vector

Assessment

Ask oral question to determine whether the given physical quantity is a scalar or vector. Also give certain time to think and list some scalar and vector on a piece of paper individually. Exchange these papers and distribute randomly. Then invite some students to read the list from the paper and decide whether the given answer is correct or not.

7.2 Representation of a Vector

Periods allotted: 3 periods

Competencies

At the end of this subunit, students will be able to:-

- represent a vector geometrically
- explain magnitude and direction of a vector

- determine magnitude and direction of vectors
- representation of a vector using column vector representation
- define parallel vectors
- identify parallel vectors

Introduction

In the previous subunit, students are familiar with scalars and vectors. In this subunit, students will discuss about representation of vectors and determine the magnitude and direction of a vector. They will also learn about different types of vectors like equal vectors and parallel vectors.

Teaching Notes

Here you can start your discussion by asking a question:- How do we represent a vector? Some students may reflect it correctly some of them may not. Since they know what a vector mean, we need a certain representation, So, inform students about the importance of representation graphical representation of vectors using examples and defining column vector in two dimension.

Answers for Exercise 7.2

1. **a.** $\vec{c}$ **b.** $\vec{b}$ **c.** $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ and $\vec{f}$
2. **a.** False **b.** False **c.** True

Answers for Exercise 7.3

1. **a.** $\vec{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

   b . $|\vec{a}| = \sqrt{1^2 + 3^2} = \sqrt{10}$, $|\vec{b}| = \sqrt{3^2 + 2^2} = \sqrt{13}$, $|\vec{c}| = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

2. $|\vec{a}| = \sqrt{(3-1)^2 + (5-2)^2} = \sqrt{13}$, $|\vec{b}| = \sqrt{(6-3)^2 + (1-1)} = \sqrt{9} = 3$

   and $|\vec{c}| = \sqrt{(7-5)^2 + (8-3)^2} = \sqrt{29}$.

Answers for Exercise 7.4

1. You can have the answer (a),(b) and (c) together.

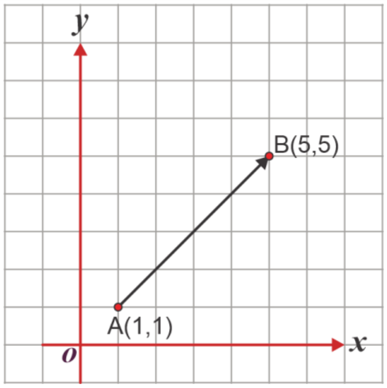


Figure 7.1

a. To determine the magnitude of this vector a student can measure the length using a ruler or use distance formula , that is

$$AB = \sqrt{(5-1)^2 + (5-1)^2} = \sqrt{32} = 4\sqrt{2} \text{ unit.}$$

b. Check it using protractor from the doted vertical line to the vector, it is $45°$.

c. The direction of $\overrightarrow{AB}$ is $N45°E$.

2.

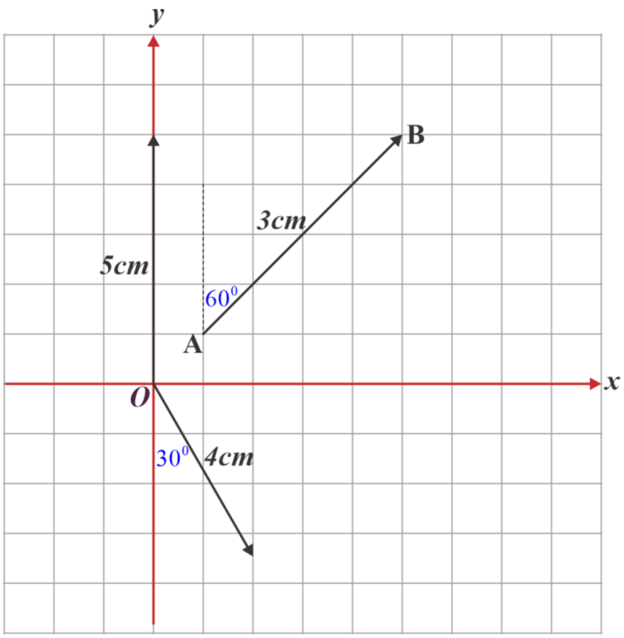


Figure 7.2

Assessment

To assess students you can use home work.

7.3 Vector Operations

Periods allotted: 5 periods

7.3.1 Addition of vectors

Periods allotted: 3 periods

Competencies

At the end of this subunit, students will be able to:-

- determine the sum of given vectors
- determine the difference of two vectors

Introduction

In the previous subunit students are able to represent and determine magnitude and direction of a vector. In this sub-unit students will study the first two operations, addition and subtraction using Triangle Law and Parallelogram Law.

Teaching Notes

A vector is a quantity which has both magnitude and direction. In subunit 7.3, students have discussed many types of vectors. So, they need to apply their understanding for operation of vectors. For example, consider parallel vectors which are conidial. their sum will be adding the magnitude and take the direction, whereas for those conidial vectors in opposite direction leads to the concept of subtraction of one vector from another. In this subunit we expect students to add and subtract vectors using triangle law and parallelogram law. The lesson will be delivered by using activities and providing illustrative examples.

Answers for Activity 7.2

1. **a.** Yes, it is possible

b. Yes, it is possible

This is just to encourage students. They can add and subtract vectors. Addition and subtraction of vectors will be delivered in the subsequent lessons.

2.

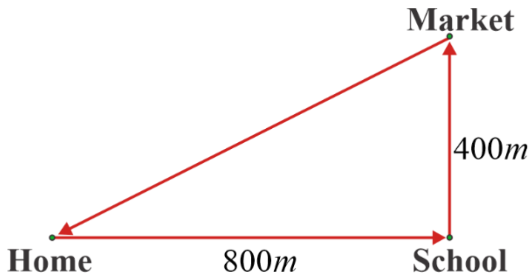


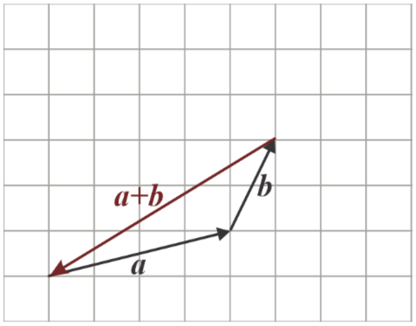
Figure 7.3

a. The student's journey is described in the above figure 7.3 above, she moved from home to school ($800\ m$) to the East, represented by $\overrightarrow{AB}$, then from school to market 400m to the North denoted as $\overrightarrow{BC}$ and finally from market to home by $\overrightarrow{CA}$.

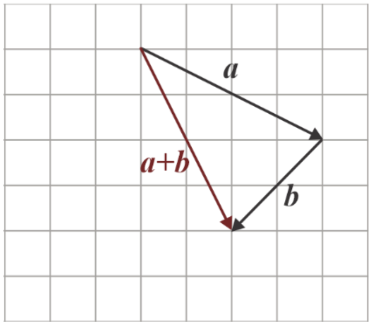
b. The magnitude of $\overrightarrow{CA}$ is determined by $CA = \sqrt{(800)^2 + (400)^2} = \sqrt{800000} = 894.42\ m$, so the total distance covered by the student is $2094.42\ m$. (Note that ΔABC is a right angled triangle.)

Answers for Exercise 7.5

(a)



(b)



Answers for Activity 7.3

1. A parallelogram is quadrilateral whose opposite sides are parallel and congruent.
2. Some of the properties of parallelogram are
   - Opposite sides are parallel and congruent ,

     For the given figure, AB is parallel and congruent to DC

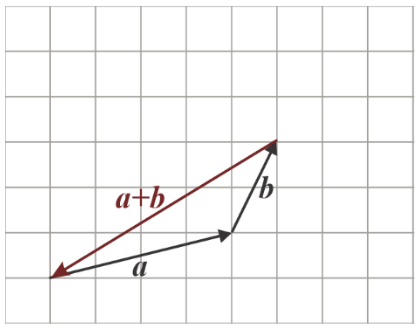
     BC is parallel and congruent to AD

     The following can also be considered as additional properties
   - The diagonal bisect each other.
   - The sum of interior angles is 360°,
   - Opposite angles are congruent
   - The diagonals are angle bisectors

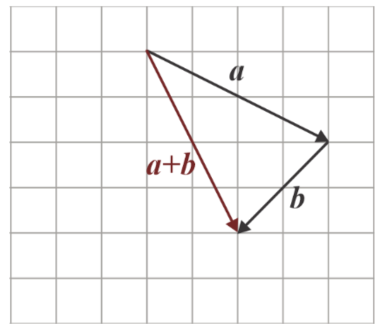
Answers for Exercise 7.6

(a) (using parallelogram law)

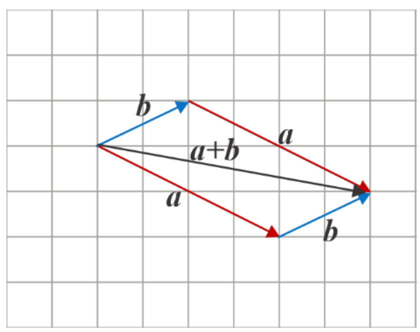
(a)

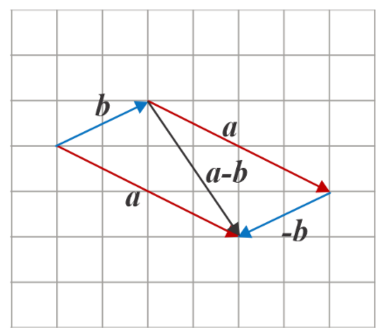


(b)



(b)





Answers for Exercise 7.7

1. Given vectors $\boldsymbol{a} = \begin{pmatrix}1\\3\end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix}2\\-1\end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix}-4\\5\end{pmatrix}$,

a) $\boldsymbol{a}+\boldsymbol{b} = \begin{pmatrix}\mathbf{1+2}\\\mathbf{3+(-1)}\end{pmatrix} = \begin{pmatrix}\mathbf{3}\\\mathbf{2}\end{pmatrix}$

b) $\boldsymbol{a}-\boldsymbol{b} = \begin{pmatrix}\mathbf{1-(-4)}\\\mathbf{3-5}\end{pmatrix} = \begin{pmatrix}\mathbf{5}\\\mathbf{-2}\end{pmatrix}$

c) $\boldsymbol{a}+\boldsymbol{c} = \begin{pmatrix}\mathbf{1+(-4)}\\\mathbf{3+5}\end{pmatrix} = \begin{pmatrix}\mathbf{-3}\\\mathbf{8}\end{pmatrix}$

d) $\boldsymbol{b}-\boldsymbol{c} = \begin{pmatrix}\mathbf{2+(-4)}\\\mathbf{-1-5}\end{pmatrix} = \begin{pmatrix}-2\\-6\end{pmatrix}$

e) $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c} = \begin{pmatrix}\mathbf{1+2-4}\\\mathbf{3-1+5}\end{pmatrix} = \begin{pmatrix}\mathbf{-1}\\\mathbf{7}\end{pmatrix}$

2. **i.** $\overrightarrow{AC} = \boldsymbol{a}+\boldsymbol{b}$ **ii.** $\overrightarrow{ED} = \boldsymbol{c}-\boldsymbol{b}$ **iii.** $\overrightarrow{BC} = \boldsymbol{a}$ **iv.** $\overrightarrow{CE} = -\boldsymbol{c}$

3 **a.** $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ or $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

b. $\overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$

c. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD}$

4

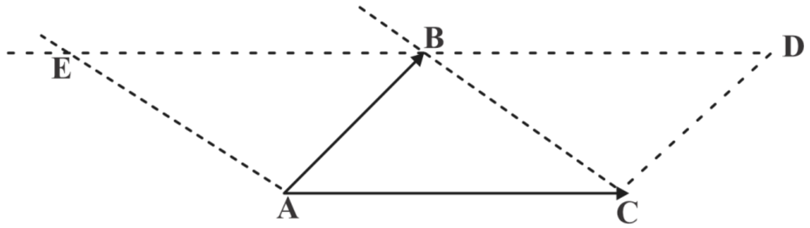


Figure 7.4

a. From the construction one can easily observe that $\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{AE}$, but $\overrightarrow{AE}$ is parallel and congruent to $\overrightarrow{BC}$. So that $\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$

b. $\overrightarrow{AD}$and $\overrightarrow{BC}$ are diagonals of a parallelogram and they can be described as $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC}$ and $\overrightarrow{CB} = \overrightarrow{AB} - \overrightarrow{AC}$.

Assessment

You can assess the students by providing different vectors on a plane and ask questions of how they can get the sum and difference of vectors graphically.

7.3.2 Multiplication of a vector by a scalar

Periods allotted: 2 periods

Competency

At the end of this subunit, students will be able to:-

- Multiplying a given vector by scalar

Introduction

In the previous subunit students are able to add and subtract vectors. In this subunit students will discuss about multiplication of a vector by a scalar. It is enlarging or squeezing the vector in the same direction or opposite direction depending on the nature of the scalar..

Teaching Notes

You need to recall about parallel vectors to start this lesson. Then, use Activity 7.4 which shows different types of parallel lines. In addition by counting the grids, students will be able to determine the magnitude of each vectors so that they can compare one vector with the other. This will lead them how one vector is expressed as a scalar multiple of the other. Then formally define scalar multiplication of a vector and supplement with appropriate example.

Answers for Activity 7.4

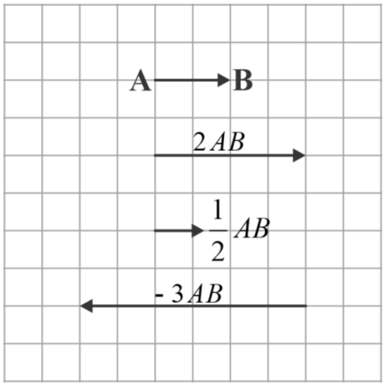
Expression of $\vec{v}$, $\vec{w}$, $\vec{n}$ and $\vec{m}$ in terms of a vector $\vec{u}$.

$$\vec{v} = 3\vec{u}, \vec{m} = -6\vec{u}, \vec{n} = \frac{1}{2}\vec{u}, and\ \vec{p} = -2\vec{u}$$

Assessment

You can ask students to enlarge or shorten the pictorial representation of a given vector quantity and let them explain the physical interpretation of enlarging or shortening a vector. These can be made in forms of class activity or group discussions. You can also use Exercise 7.4 to assess your students.

Answers for Exercise 7.8



Answers for Exercise 7.9

1. Given vectors $\boldsymbol{a} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

   a. $4\boldsymbol{a} = 4\begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ 16 \end{pmatrix}$

   b. $-3\boldsymbol{b} = -3\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} -9 \\ -18 \end{pmatrix}$

   c. $\frac{1}{3}\boldsymbol{b} = \frac{1}{3}\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

   d. $-\frac{1}{3}\boldsymbol{b} = -\frac{1}{3}\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

   e. $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$

2. Write true if the statement is correct and false otherwise.
   a. True
   b. False (for k between 0 and 1, the magnitude is less than $\vec{u}$)
   c. False(they may be opposite in direction)
   d. True
3. From the following figure 7.30, describe each vector as as a scalar multiple of $\overrightarrow{AB}$.

   $\overrightarrow{AF} = -\frac{1}{2}\overrightarrow{AB}, \overrightarrow{AC} = \frac{5}{2}\overrightarrow{AB}, \overrightarrow{AD} = 4\overrightarrow{AB}, and\ \overrightarrow{AG} = -2\overrightarrow{AB}.$

7.4 Position Vector

Periods allotted: 2 periods

Competencies

At the end of this subunit, students will be able to:-

- express any given vector as a position vector
- determine components of a vector

Introduction

Up to now in the previous 4 subsections of this unit, students represent vectors by taking the initial and terminal points randomly, we call such vectors are free vectors. They have studied how to operate such vectors. In this sub unit they will learn another way of representing vectors by fixing the initial at a certain point called origin. This will simplify operation of vectors and help to determine the magnitude and direction of a vector easily. Students also discuss the use of position vector to relate vectors with trigonometry and its advantage to solve application problems.

Teaching Notes

The main objective of this subunit is to represent any vector as a position vector. Students are expected to use ruler and compass and should be able to construct parallel lines. To review their previous understanding and initiate about the next lesson give activity 7.5 as a group work.
Make a group based on their seat, and reflect their solution. After this activity, you can discuss the definition for the position vector.

Answers for activity 7.5

1. a. Vectors $\vec{p}$ and $\vec{q}$ are parallel.

b .Vectors $\vec{v}$ and $\vec{w}$ are coinitial.

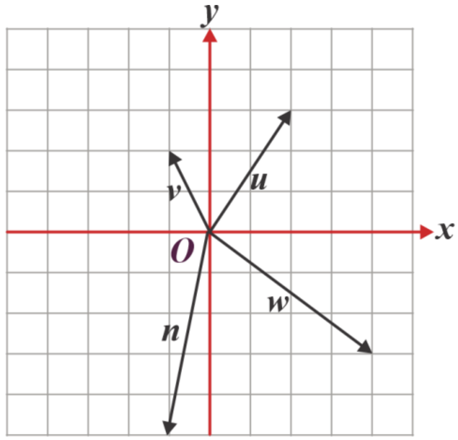
c. Yes, we can

d . Vectors $\vec{v}$ and $\vec{w}$ start from the same point which is the origin.

2. a) To move the initial point $A = (1,2)$ to $(0,0)$, we add the component $(-1,-2)$ to the components of A. So that we add the same component $(-1,-2)$ to the terminal point too. Therefore, the terminal point B moves to point C, and it is denoted by the order pair $(3,5) + (-1,-2) = (2,3)$.

b) The two vectors $\overrightarrow{AB}$ and $\overrightarrow{OC}$ are parallel, have the same magnitude and direction.

Answers for Exercise 7.10



The magnitude of each vector will be $|\vec{u}| = \sqrt{2^2 + 3^2} = \sqrt{13}$, $|\vec{v}| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$,

$|\vec{w}| = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$ and $|\vec{n}| = \sqrt{(-1)^2 + (-4)^2} = \sqrt{17}$

Answers for Exercise 7.11

1. i. We need to determine the position vector of $\overrightarrow{BA}$. The corresponding position vector is

$$\overrightarrow{OP} = (-5 - 1, 1 - 4) = (-6, -3).$$

ii. Similarly, the position vector corresponding to $\overrightarrow{DC}$ will be

$$\overrightarrow{OQ} = \big(-3 - 5, 4 - (-1)\big) = (-8, 5).$$

2. i. The vector $\overrightarrow{BC}$ is horizontal line with magnitude 4 units to the East

ii. The magnitude of $\overrightarrow{\boldsymbol{AB}}$ can be calculated using its corresponding position vector as $|\overrightarrow{\boldsymbol{AB}}| = \sqrt{(-\mathbf{6})^2 + (-\mathbf{3})^2} = \mathbf{3}\sqrt{\mathbf{5}}$ units. Regarding to its position, we use tangent on the position vector corresponding to $\overrightarrow{\boldsymbol{BA}}$. That is, $\tan\boldsymbol{\theta} = \frac{\mathbf{6}}{\mathbf{3}} = \mathbf{2}$, where $\boldsymbol{\theta}$ is the angle between the vector and the $\boldsymbol{y} - \boldsymbol{axis}$. Using trigonometric table we got $\boldsymbol{\theta} \approx \mathbf{63.43}°$. Since the terminal point is located in the 4th quadrant its position is $.\mathbf{43}°\boldsymbol{W}$. Hence, $\overrightarrow{\boldsymbol{AB}}$ is a vector with magnitude $\mathbf{3}\sqrt{\mathbf{5}}$ units in the direction $\boldsymbol{S}\mathbf{63.43}°\boldsymbol{W}$. You can also use graphs.

Assessment

You can assess the students by asking questions to determine the magnitude and direction of a vector using components. That can be done through assignment or test.

7.5 Applications of Vectors in Two Dimensions

Periods allotted: 2 periods

Competency

At the end of this subunit, students will be able to:-

- solve problems related to vectors

Introduction

Students studied how to represent vectors, how to determine magnitude and direction of a vector, operation on vectors (addition, subtraction and multiply a vector by scalar) in the previous sub units. In this subunit, they are expected to solve problems related to vectors.

Teaching Notes

This subunit is designed to discuss the application of vectors and solve problems. So select different problems to the class and students will solve using the basics they have from vector and trigonometry lessons

Answers for Exercise 7.12

1. **a.** $\boldsymbol{a} + \boldsymbol{b}$ **b.** $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$ **c.** $\overrightarrow{AF} + \overrightarrow{FD} = \overrightarrow{AD} = \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$

   d. $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = \boldsymbol{b} + \boldsymbol{c}$
2. The problem is described as in figure 7. 5 , where the force along with the rope is 100N. What we need is the component of this vector, F_x and F_y.

To get their magnitude we use Pythagoras theorem. The horizontal component is $\sin 60° = \frac{F_y}{100N}$ so that $F_y = 100 \times \sin 60°\ N \approx 100 \times 0.87\ N = 87N$

And the vertical component is $\cos 60° = \frac{F_x}{100N}$ so that $F_x = 100 \times \cos 60°\ N = 50N$. Since the direction is not described in the equation, the direction of F_x could be to the right or to the left, but the direction of F_y is to upward in both cases.

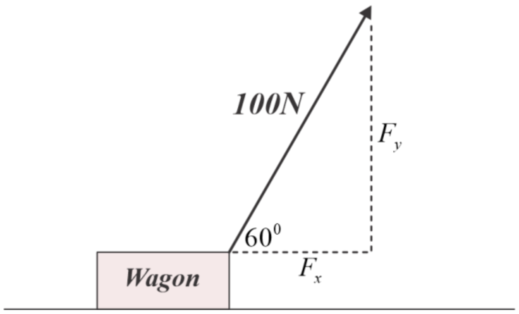


Figure 7.5

3. The river flows at the right as shown in figure 7.6. The swimmer starts from point A and moves to point C due to the river flow influence. Now we need to determine the position of the swimmer, that is AC and θ. Since we have right angled triangle, we can apply Pythagoras theorem to determine magnitude of AC.

   That is $AC = \sqrt{80^2 + 40^2} = 40\sqrt{5}m$. Using tangent we can determine θ.

   $\tan\theta = \frac{40m}{80m} = \frac{1}{2}$. Hence, $\theta \approx 27°$. So the swimmer is located $40\sqrt{5}\,m$ to $N27°E$.

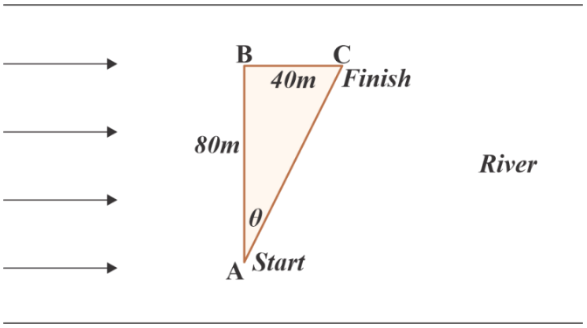


Figure 7.6

Summary

Revise the basic concepts in the unit orally. Summarize the main points of the unit. Focus on operation of vectors and different types of vectors. To check whether they understood or not provide the review exercise. Arrange a tutorial class and observe their reflection on the review exercise.

Answers for Review Exercise

1. Scalars are quantities that are fully described by a magnitude (or numerical value) alone
   Vectors are quantities that are fully described by both a magnitude and a direction.
2. Scalars:- -temperature, density, area, distance, time, mass, …
   Vectors:- displacement, force, pressure, acceleration, velocity, …
3. The magnitude of the vectors will be:
   $|\vec{u}| = 4, |\vec{v}| = 2, |\vec{w}| = 3\sqrt{2}, |\vec{p}| = 4, |\vec{q}| = 3\sqrt{2}$
4. $\overrightarrow{AD} = (14, -3), \overrightarrow{BC} = (3, -2), \overrightarrow{CD} = (3, -6) and\ \overrightarrow{AB} = (8,5).$
5. **a.** $\boldsymbol{g}$ **b.** $\boldsymbol{c}$ **c.** $\boldsymbol{g}$ **d.** $\boldsymbol{f}$
6. **a.** $\overrightarrow{AD}$ **b.** $\overrightarrow{PN}$ **c.** $\overrightarrow{LK}$
7. **i.** $-\boldsymbol{c}$ **ii.** $\boldsymbol{d}$ **iii.** $\boldsymbol{f}$ **iv.** $\boldsymbol{e}$ **v.** $\boldsymbol{g}$
8. The component of the vector form for $\overrightarrow{AB} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ and its magnitude is
   $|\overrightarrow{AB}| = \sqrt{0^0 + 2^2} = 2$ also $\vec{u} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$ $and\ its\ magnitude\ is\ |\vec{u}| = \sqrt{6^2 + 8^2} = 10.$ So that we can compare the magnitudes as of $|\vec{u}| = 5|\overrightarrow{AB}|$.
9.

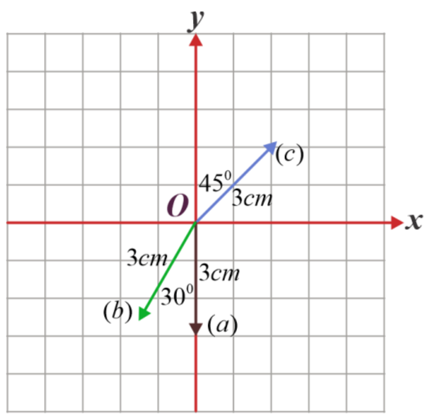


Figure 7.7

10. Given that $A = 3i - 2j$ and $B = 5i - \frac{1}{2}j$ and we need to determine $\frac{1}{2}A - \frac{2}{3}B$, that is
    $\frac{1}{2}(3i - 2j) - \frac{2}{3}\left(5i - \frac{1}{2}j\right) = \left(\frac{3}{2}i - \frac{10}{3}i\right) + \left(-j + \frac{1}{3}j\right) = -\frac{11}{6}i - \frac{2}{3}j.$
    So that the answer is 'C'.

11. For the given figure 7.8, PQRS is a parallelogram and $\overrightarrow{PT} = -\overrightarrow{QS}$. We need the one which is not true.

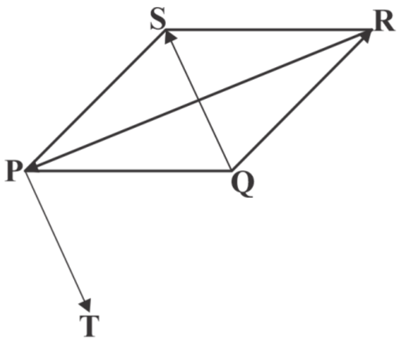


Figure 7.8

a. $\overrightarrow{PR} - \overrightarrow{QR} = \overrightarrow{PR} + \overrightarrow{RQ} = \overrightarrow{PQ} \neq \overrightarrow{QP}$ is not true

b. $\overrightarrow{PQ} - \overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QT} = \overrightarrow{PT}$ which is true.

c. $\overrightarrow{PS} - \overrightarrow{PT} \neq \overrightarrow{RS}$

d. $\overrightarrow{PS} - \overrightarrow{SR} \neq \overrightarrow{PR}$

Hence the answer is 'B'

12. The vector is $= 5i + j$, we need equal vector to V.

a. $\overrightarrow{EF} = (0 + 1{,}3 + 2) = i + 5j$

b. $\overrightarrow{GH} = (2 - 3{,}2 + 1) = -i + 3j$

c. $\overrightarrow{PQ} = (2 + 3, -1 + 4) = 5i + 3j$

d. $\overrightarrow{RS} = (10 - 5{,}7 - 6) = 5i + j$

So the correct answer is 'D'.

13. Given that $\vec{v} = \overrightarrow{PQ}$ where $P = (1{,}0)$ and the point Q lies on the $y - axis$ .So that its coordinate has a form $Q = (0, x)$ where x to be determined. Also $|\vec{v}| = \sqrt{10}$. Now let us determine x using magnitude and components of $\vec{v}$.

$\sqrt{10} = \sqrt{1^2 + x^2} \Rightarrow x = 3$ (it is positive since it was given Q is located above P). Hence, the point $Q = (0{,}3)$.

14. Given that $\vec{a} = -2\vec{\imath} - 4\vec{\jmath}$ also $\vec{b} = (-1 - 1{,}4 - 2) = (-2{,}2) = -2\vec{\imath} + 2\vec{\jmath}$. So that we can find $-3\vec{a} + \vec{b} - 4\vec{\imath} + \vec{\jmath} = -3(-2\vec{\imath} - 4\vec{\jmath}\,) + -2\vec{\imath} + 2\vec{\jmath} - 4\vec{\imath} + \vec{\jmath} = 0\vec{\imath} + 15\vec{\jmath}$

Hence, the magnitude of $-3\vec{a} + \vec{b} - 4\vec{\imath} + \vec{\jmath}$ is 15.

15. We can represent the journey of the merchant graphically as shown in figure 7.9 below. What we are asked is to determine the position of the merchant (H) relative to his starting point (G). The magnitude of $\overrightarrow{GH}$ will be $|\overrightarrow{GH}| = \sqrt{6^2 + (12)^2} = 6\sqrt{5}km$.

Using trigonometric ratio $\tan\theta = \frac{FH}{GH} = \frac{6}{12} = 0.5$,

using trigonometric table, we get $\theta \approx 26.57°$.

Hence, the merchant is located $6\sqrt{5}$km, in the direction $N26.57°E$.

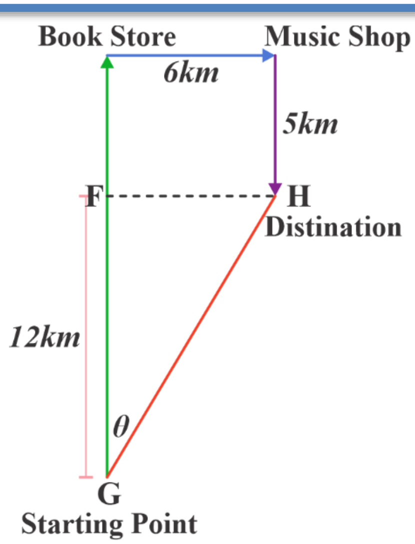


Figure 7.9

16. We can see the result using the following diagram

We can have the following options for the resultant vector depending to the size (magnitude of $\vec{u}$ and $\vec{v}$).This is to mean the resultant vector may be to the north direction.

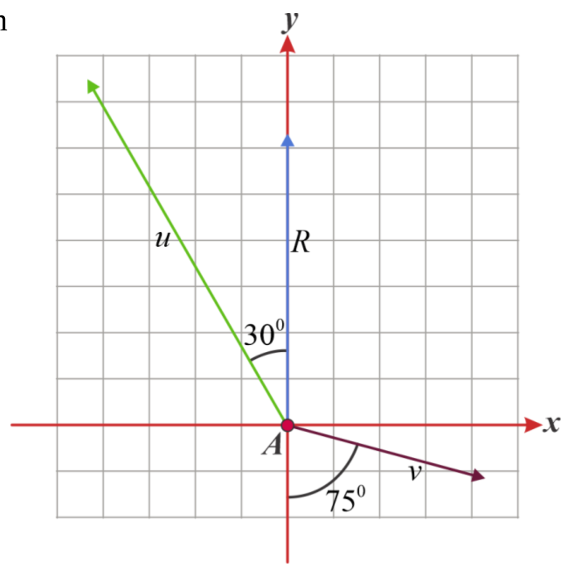


Figure 7.10

17. We use the following graphs to express the trip of the airplane. Let the plane start at the point A and terminate at B as shown in the figure 7.11 (a) below. This first trip covers $100\ km$. Then, the next trip is from B to C which covers $150km$.

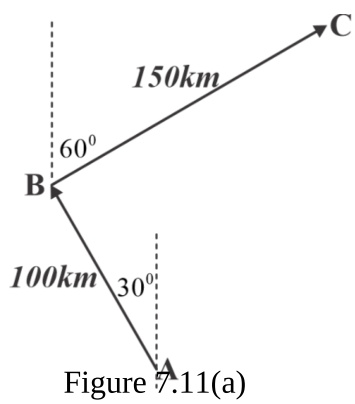


Figure 7.11(a)

We cannot add the two vectors directly and obtain the resultant. First we will determine the x and y components for each vector $\overrightarrow{AB}$ and $\overrightarrow{BC}$ as shown in figure 7.11(b) and figure 7.11(c) below.

$x - component$	$y - component$
$\sin 30° = \frac{x_1}{100km}$	$\cos 30° = \frac{y_1}{100km}$
$x_1 = 100km \times \sin 30°$	$y_1 = 100km \times \cos 30°$
$x_1 = 50km$	$y_1 \approx 87km$

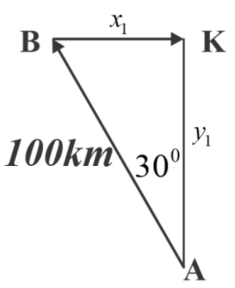


Figure 7.11(b)

$x - component$	$y - component$
$\sin 60° = \frac{x_2}{150km}$	$\cos 60° = \frac{y_2}{150km}$
$x_2 = 150km \times \sin 60°$	$y_1 = 150km \times \cos 60°$
$x_1 \approx 130km$	$y_2 = 75km$

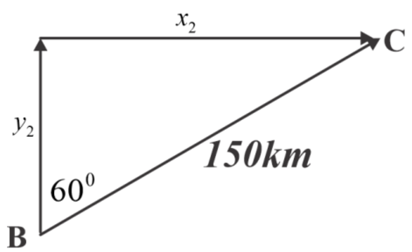


Figure 7.11(c)

From fig. 7.11(b) and fig.7.11(c), we can observe that both $y - components$ are pointing up, so we will call both of them positive. When we add them, we get

$87km + 75km = 162km$ (which is also positive and pointing up).

The $x - components$ are little different. The first one (x_1) points to the left, so we will call it negative and the second (x_2) points to the right so it is positive. That is

$-50km + 130km = 80km$ (to the right).

Now we will draw the new diagram (figure 7.11(d)) with these components of x and y to determine the resultant.

Using Pythagoras theorem

$(AC)^2 = (AD)^2 + (DC)^2$

$AC = \sqrt{(162)^2 + (80)^2}$

$AC = 181km$ and $\tan\theta = \frac{80km}{162km} \Rightarrow \theta \approx 26.28°$.

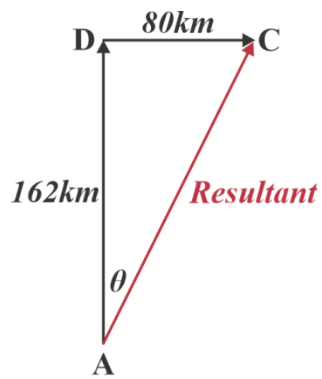


Figure 7.11(d)

Hence , the plane displacement is 181km in the direction of N26.28°E.

UNIT 8

STATISTICS AND PROBABILITY (30 periods)

INTRODUCTION

Statistics deals with data; its importance has been realized by governments, by the private sector, and throughout disciplines because of the need for data-based decision making. It has become even more important in the past few years. These days, more and more data is being collected, stored, analyzed and re-analyzed. Therefore, students are required to have some of the fundamentals about statistics from their primary grades mathematics.

In this unit, students will get acquainted with basic ideas of statistics and probability. In statistics, students will be brought into many new terms like, descriptive statistics, population, population function, primary data, secondary data, frequency distribution table, etc. They will also practice constructing frequency distributions and their Histograms.

In this unit, students are required to be familiar with measures of central tendency such as Mean, Median and Mode, and some of the measures of dispersion such as Range, Variance and Standard deviations. Eventually, the students will be brought into the notations of experiment, sample space (or possibility set), event and probability of an event.

After completing this unit, students will be able to:

- collect and represent simple statistical data using different methods such as histograms.
- understand facts and basic principles about probability
- develop the concept of probability via experimentation and hypothetical events

Teaching aids that could be used in Unit 8

Presentations and representations of different data need important teaching aids so that it alleviates problems in teaching learning. Moreover, teaching aids make things clear. In this respect, consider the following: Graphs like, histograms, bar charts, pie charts, line graphs which are also useful for comparing, dice and different coins for probability. You may employ different statistical soft wares for describing different graphs and calculating diverse statistics.

8.1. Statistical Data

Periods allotted: 16 Periods

Competencies

- differentiate primary and secondary data.
- collect data from their environment.
- classify and tabulate primary data according to the required criteria.
- construct a frequency distribution table for ungrouped data.
- collect and represent simple statistical data using different methods such as histograms and interpret.
- determine the mean, median and mode(s) of a given data.
- describe the purposes and uses of mean, median and mode.
- identify the properties of the mean of a given data (population function).
- compute the measures of dispersion for ungrouped data.
- describe the purpose and use of measures of dispersion for ungrouped data.

Key words: Primary and secondary data, Data collection, Quantitative and Qualitative data, Descriptive and Inferential Statistics, Frequency, distribution, frequency distribution and Histogram, Mean, Median, Mode, Range, Standard deviation, and Variance.

8.1.1 Collection and tabulation of statistical data

The data collected for the use of statistical research sometimes comprises a few reasonably simple figures, which can be easily realized without any particular discourse. However, more often there is a very intense mass of raw data without any form. Thus, unmanageable, unorganized and formless mass collected is not capable of being quickly or easily associated or understood. Unorganized data are not suitable for further analysis and interpretation. In order to make the data easy and simply understandable the first activity is not condense and simplify them in such a way that irrelevant data are removed and their significant characteristics are excel prominently. The process followed for this role is known as method of classification and tabulation. Classification helps proper tabulation.

To start with, you can group the students and let them do **Activity 8.1**. The purpose of this group work is enlightening and sensitizing students on the use and application of statistics. It will also give them the chance of practicing data collection, organization and some level of interpretation.

The group work will also give ideas on how to collect data from our environment and make observations after the students collect data. They can discuss how to get information from the data collected and, in the meantime, they can discuss the importance of statistics in different fields.

Answers to activity 8.1

a. The solutions for this activity depend from school to school. You are required to do is, however, help discussion on the results students bring. The students can classify and present the data they collected.

b. The solutions for this activity also depend from school to school. You are required to do is, however, help discussion on the results students bring. The students can classify and present the data they collected.

c. The same as b and c

In this sub-section, we have experienced what statistics is and its usage in different fields. The students have also exercised collecting and tabulating data.

Answers to activity 8.2

The solutions for this activity to each question depend from school to school and hospital to hospital. You are required to do is, however, help discussion on the results students bring. The students can classify and present the data they collected.

Assessment

You can give practical assignments and activities to assess or give exercise 8.1 as a group work and students present their work.

Answers to exercise 8.1

b, c and d are quantitative data whereas a) and e) are qualitative data.

Answers to exercise 8.2

a)

$Value$	$frequency$
1	3
2	1
3	2

4	4
5	2
6	3

b)

Class interval (v)	frequency
$0 \le v < 5$	3
$5 \le v < 10$	3
$10 \le v < 15$	4
$15 \le v < 20$	2

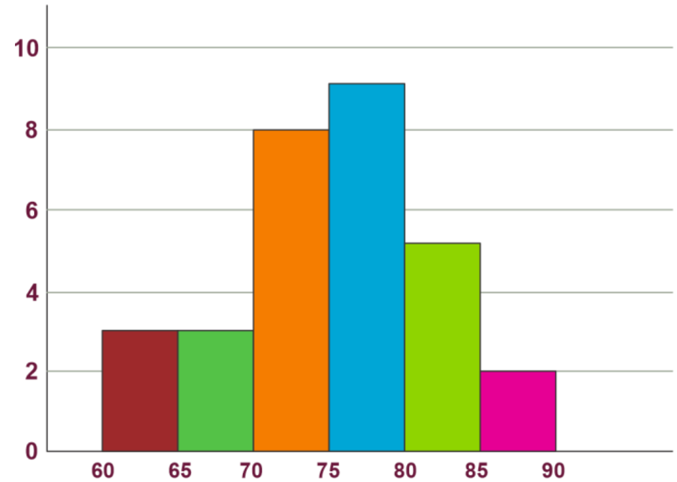
8.1.2 Graphical presentations of statistical data

When you deal with very large amounts of data, for instance, in a typical survey, by the completion of your data collection phase you may have accumulated thousands of individual responses represented by a scramble of numbers. To make sense out of these data, you will have to organize and summarize them in some systematic fashion. The most basic method for organizing data is to classify the observations into a frequency distribution. A frequency distribution is a table that reports the number of observations that fall into each category of the variable we are analyzing. Constructing a frequency distribution is usually the first step in the statistical analysis of data. You try to construct and interpret histogram, a pie chart, bar graph and line graph.

Answers to activity 8.3

Use continuous data, equal class-interval, (size) and class-limits

Class interval	frequency
$60 \le \mathrm{h} < 65$	3
$65 \le \mathrm{h} < 70$	3
$70 \le \mathrm{h} < 75$	8
$75 \le \mathrm{h} < 80$	9
$80 \le \mathrm{h} < 85$	5
$85 \le \mathrm{h} < 90$	2

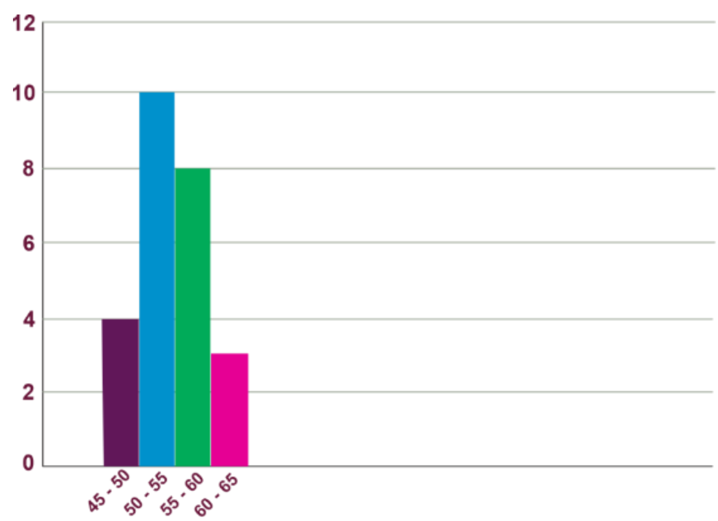


Answers to Exercise 8.3

Help students to construct histograms using statistical tools such as SPSS.

1. Construct a histogram for the following frequency distribution table that describes the frequencies of weights of 25 students in a class.

Weight (w) in kgs	Frequency(Number of students)
$45 \leq w < 50$	4
$50 \leq w < 55$	10
$55 \leq w < 60$	8
$60 \leq w < 65$	3

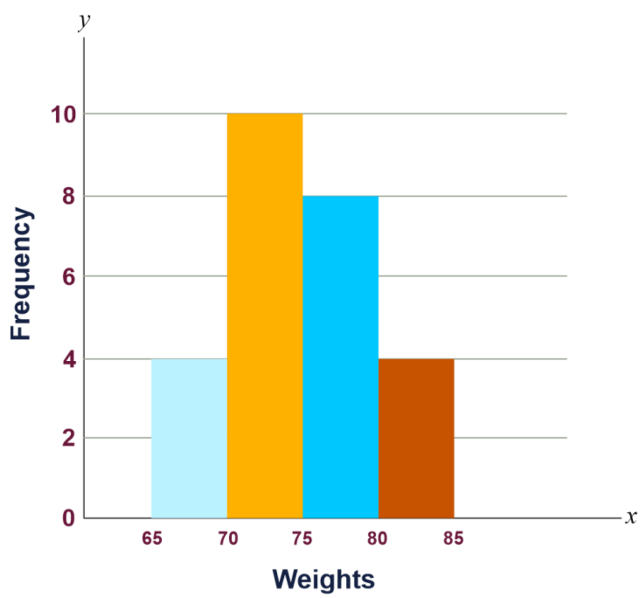


Steps to draw a histogram:

Step 1: On the horizontal axis, we can choose the scale to be $1\ unit = 11\ lb$. Since the weights in the table start from 65, not from 0, we give a break/kink on the X-axis.

Step 2: On the vertical axis, the frequencies are varying from 4 to 10. Thus, we choose the scale to be $1\ unit = 2$.

Step 3: Then draw the bars corresponding to each of the given weights using their frequencies.



8.1.3. Measures of central tendency

Although it is important to teach students how to crunch numbers to find the mean or the standard deviation of a data set, teaching the process of calculation does little to help students grasp important statistical concepts. Instead, instructors should supplement lessons in calculation with instruction in the underlying idea, according to the International Conference on Teaching Statistics. In a unit about measures of central tendency like mean and median, for example, you might ask students to guess mean and median income in their town. When the mean increases exponentially, the median may remain unchanged. Use this as a starting point to discuss which measure is more appropriate for different problems.

Answers to activity 8.4

Use the formula for mean, median and mode (the most frequent value), collect identical values to construct the table.

a.

Score (25%)	9	10	11	12	13	14	18	21	
frequency	3	1	2	4	1	5	3	1	20

b. 13.25

c. 11 students score below average, No one score on average and 9 students score above the mean.

d. It is possible to say all about their performance using the mean. Only 45% scored more than the mean. So, the students are weak.

e. 12.5

f. 14.

Answers to Exercise 8.4

a) $\bar{x} = 60$

b) $\bar{x} \cong 165.83$

Answers to activity 8.5

Use the formula to find the mean each of the questions.

a. 50 **b.** 0 **c.** 55 **d.** 45 **e.** 250 **f.** In general, in b), the sum of the differences of each value from the mean is zero. In c, the new mean is greater than the old mean by 5. In d, the new mean is less than the old mean by 5. In e, the new mean is 5 times the old.

Answers to Exercise 8.5

Help students to use statistical tools such SPSS, IT or calculator to check their answer.

1 **a)** the new mean is $4 + 1 = 5$

b) the new mean is $4 - 2 = 3$

c) the new mean is $(4)(3) = 12$

d) the new mean is $\frac{4}{2} = 2$

2. 110

3. 2,000

Answers to Exercise 8.6

a) The median is 5

b) $\bar{x} = \frac{181}{9} \cong \mathbf{20.11}$ and the median is 10

Answers to Exercise 8.7

a) The mode is the most frequent value, which is 5

b) The mode(s) is/are the most frequent value(s), which are 13 & 16. The data is called bi-modal.

c) The mode is 13

Answers to Exercise 8.8

1.

a. The mean number of pages read in five years is 18.

b. The median is 16

c. No repeated value. Hence no mode.

2. **a)** The mean is approximately 88.167 kg and **b)** the middle value is 88.5

3. No, we cannot as they are not quantities.

4. The solutions for this activity depend from town to town and school to school. You are required to do is, however, help discussion on the results students bring. The students can classify and present the data they collected.

5. The mean is 62, the median is 70 after arranging in increasing order $23, 26, 32, 32, 59, 70, 82, 83, 87, 94, 94$ and the mode is 32 and 94.

6. **a)** The mean is 0.45, the median is 1 and the mode is the most frequent value, which is 1.

b) Six of them are greater than or equal to 2.

7. $\frac{\mathbf{2+3+x+5+6+12}}{\mathbf{6}} = 5$. Then, $x = 2$.

8. k + b
9. y = 7.5
10. In an examination, the mean of marks scored by a class of 40 students was calculated as 72.5. Later on, it was detected that the marks of one student were wrongly copied as 48 instead of 84. Find the correct mean.

Ans: **Incorrect Mean of** marks $= \frac{\text{Incorrect sum of 40 students}}{40} = 72.5$

Incorrect sum of marks of 40 students $= (72.5)(40) = 2900$

Since the marks of one student were wrongly copied as 48 instead of 84, Correct sum of marks of 40 students $= 2900 - 48 + 84 = 2936.$

So, the correct mean $= \frac{2936}{40} = 73.4$

11. The new mean is 1465

Answers to activity 8.6

Consider the following four sets of numbers representing ages of children in a village. Try to find the mean in each case and compare it to the different values and find the variation.

1. Complete the table by finding the sum of each group and the mean, median and mode.
2. All the means are not equal. However, Groups I, II and IV have the same mode.

Group	Values										Mean
I	5	7	6	7	4	5	4	6	8	5	5.7
II	2	1	4	3	9	6	3	2	4	3	3.7
III	6	5	7	5	6	7	5	6	5	7	5.9
IV	5	5	5	5	5	5	5	5	5	5	5.0

3. Mean deviations
   i. The mean of Group IV is closest to each value (of course each value is equal to the mean)
   ii. The greatest difference between the mean and each value exists in Group II

Group	Values										Mean
I	-0.7	1.3	0.3	1.3	-1.3	-0.7	-1.3	0.3	2.3	-0.7	5.7
II	-1.7	-2.7	0.3	-0.7	6.3	2.5	-0.7	-1.7	0.3	-0.7	3.7
III	0.1	-.9	1.1	-.9	.1	1.1	-.9	.1	-.9	1.1	5.9
IV	0	0	0	0	0	0	0	0	0	0	5.0

4. The group that shows most variation is group II. The group that shows slight variation are groups I & III. Group IV does not show any variation.
5. The range for each group: group I; $8-4=4$, Group II; $9-1=8$ Group III; $7-5=2$; Group IV; $5-5=0$

Answers to Exercise 8.9

a) Range = Largest value – Smallest value $= 85-35=50$

b) Range = Largest value – Smallest value $= 54-5=49$

Answers to activity 8.8

First find the mean and then follow each steps $b, c, and\ d$

a. 6

b. $4-6=-2,\ 5-6=-1, 7-6=1, 8-6=2, 7-6=1$ and $5-6=-1$

c. 4, 1, 1, 4, 1, 1

d. The mean of the squared deviation is $\frac{4+1+1+4+1+1}{6}=2$ and its principal square root is

$$\sqrt{\frac{4+1+1+4+1+1}{6}}=\sqrt{\frac{12}{6}}=\sqrt{2}$$

e.

Answers to Exercise 8.10

1. a) $\bar{x}=5$ b) $\sigma^2 \cong 6.67$ c) $\sigma \cong \sqrt{6.67}$
2. a) $\sigma^2=3.2$ b) $\sigma=\sqrt{3.2}\cong 1.79$

Answers to Exercise 8.11

x_i	f_i	$x_i f_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
1	6	6	$1 - 4 = -3$	9	54
2	9	18	$2 - 4 = -2$	4	36
3	4	12	$3 - 4 = -1$	1	4
4	1	4	$4 - 4 = 0$	0	0
	$N = 20$	$\sum x_i f_i = 40$		$\sum (x_i - \bar{x})^2 = 14$	$\sum f_i(x_i - \bar{x})^2 = 94$

Answers to activity 8.9

First calculate the mean and then compute the formula of variance and standard deviation and compare them in the next five days:

1. The mean is 50
2. The variance is 520. So, the standard deviation is $\sqrt{520}$
3. **a.** 55 **b.** The variance is 520. So, the standard deviation is $\sqrt{520}$ **c.** The mean differs by 5; however, the variance and standard deviation are the same. **d.** Adding equal amount changes the mean by that amount but does not change the variance and standard deviations.

Answers to Exercise 8.12

a) $\sigma^2 = 6.8,\ \sigma = \sqrt{6.8} \cong 2.61$

b) The new variance = the old variance and the new standard deviation = the old standard deviation.

c) The new variance = the old variance and the new standard deviation = the old standard deviation.

Answers to Exercise 8.13

a) $\sigma^2 = 38.5,\ \sigma = \sqrt{38.5} \cong 6.2048$

b) The new variance $154 = 2^2$ times the old variance. i.e $2^2(38.5) = 154$ and the new standard deviation $=$ 2 times the old standard deviation. i.e $2\sqrt{38.5} \cong 12.4096$

c) Similarly, the new variance $= \left(\frac{1}{7}\right)^2$ *times* the old variance and the new standard deviation $= \frac{1}{7}$ *times* the old standard deviation.

Answers to Exercise 8.14

1. $\bar{x} = 6.67$ the median is 7 and the modes are 5 and 9
2. Range = 6-1 = 5, variance = 2 and standard deviation $= \sqrt{2}$
3. Range $= 3 - (-3) = 6$, variance = 3.6 and standard deviation $= \sqrt{3.6}$
4. mean $= \frac{5+y+5+5+5}{5} = \frac{y+20}{5}$. Then calculate the deviation of each value from the mean . Then square all the deviations from the mean then add all of them and divide by 5. This equal to the standard deviation (= 5) square. Finally, solve for y.
5. **a.** The variance remains the same because the same amonut is added to each value

   b. the standard deviation $= \sqrt{m}$

   c. the standard deviation $= a^2\sqrt{m}$
6. **a.** The mean will be tripled and the variance $4680 = (3^2)(\text{ the original variance}) = (9)(520)$. Then, the standard deviation is $\sqrt{4680}$
7. $\sigma^2 = 11.43; \sigma = \sqrt{11.43}$
8. $\sigma^2 = 1.7225; \sigma = \sqrt{1.7225}$

8.2. Probability

Periods allotted: 14 Periods

Competencies

- Determine the probability of an event from a repeated experiment.
- Determine the probability of an event.

Key words: experiment, sample space, event, probability

Introduction

It is usual to relate achievement or unluckiness in one element with success or unlucky. Looking to decide how possibly something can arise is useful in applications. For such a purpose, discussing opportunity is vital which will help constitute such concepts as risk or luck mathematically. In this sub-unit, students will discuss introductory standards of chance starting from terminologies to opinions of probability that consist of relative frequency approach and axiomatic method to opportunity.

Before you start the sub-unit, ask the students to answer different class activities. Tell the students to read each item carefully, solve if needed. This will help assess learner's prior knowledge, skills and understanding of mathematical concepts related to probability.

Answers to Activity 8.10

1. Which of the following is different from the others?

a. Chance **b.** Interpretation **c.** Possibilities **d.** Uncertainty.

b is the answer as all inform us about probability

2. The answer is $\frac{3}{4}$

Answers to Exercise 8.15

1. **a)** $\frac{2}{4} = \frac{1}{2}$ **b)** $\frac{1}{4}$

2. **a)** $\{H, T\}$ **b)** $\{HHH, HHT, HTH, HTT, THT, TTH, THH, TTT\}$

Answers to Activity 8.11

1. Yes, because we know all the possibilities

2. $\{1,2,3,4,5,6\}$

3. Getting the number 3 on the upper face of the die.

4. **a.** The event does not happen. **b.** the event always happen (certain)

c. a is impossible and b is certain.

5. **i.** One possibility out of the six **ii.** No possibility **iii.** Four possibilities out of the six

Answers to Exercise 8.16

i. $S = \{3, 4, 5, 6, \dots \dots .30\}$

ii. **a)** $S = \{5, 6, \dots \dots .30\}$ **b)** $S = \{3,4,5, \dots \dots 15\}$

Answers to Activity 8.12

Record the number of times the coin is tossed and identify the heads and tails.

The number of record depends from the number of trails to trials. As the number of trails increases, the theoretical value of probability tends to be closer to the actual value.

Answers to Exercise 8.17

1. a) i) 4390 **ii)** $\frac{4390}{10,000} = \frac{439}{1000} = 0.439$

2. b) i) 20 times **ii)** $\frac{16}{100} = 0.16$ **iii)** $\frac{18+17}{100} = \frac{35}{100} = 0.35$ **iv)** $\frac{20+15+18}{100} = \frac{53}{100} = 0.53$

Answers to Exercise 8.18

1. $\frac{1}{2}$ **2.** $\frac{2}{5}$ **3.** $\frac{3}{10}$ **4.** $\frac{1}{10}$

Answers to Exercise 8.19

a) $\frac{3}{8}$ **b)** $\frac{1}{8}$ **c)** $\frac{1}{2}$

Answers to Exercise 8.20

1 b) $\frac{1}{4}$ **c)** $\frac{1}{6}$

2 b) $\frac{1}{3}$ **c)** 0 **d)** 0

Answers to Exercise 8.21

Help students to check their answer using SPSS or IT whenever necessary.

1. **a)** $s = \{1,2,3,4,5,6\}$ **b)** $E = \{1,2,3\}$
2. $\frac{1}{4}$
3. Two dice are simultaneously thrown once. All the possible elements for each event are:

   a. $\{(2,6),(6,2),(3,5),(5,3),(4,4)\}$

   b. $\{(2,1),(1,2)\}$

   c. $\{(1,2),(2,1),(4,2),(2,4),(6,3),(3,6)\}$

   d. $\{(1,2),(2,1),(2,3),(3,2),(3,4),(4,3),(4,5),(5,4),(5,6),(6,5)\}$

4. Refer to the example in the textbook
5. $\frac{20}{50} = \frac{2}{5}$
6. a. $\frac{6}{15} = \frac{2}{5}$ b. $\frac{4}{15}$ c. $\frac{5}{15} = \frac{1}{3}$
7. i. Total number of the sample space is equal to 36.

   So, the probability $\frac{\text{Number of event}}{\text{number os sample space}} = \frac{16}{36} = \frac{4}{9}$
8. $\frac{12}{36} = \frac{1}{3}$

Assessment

you can assess your students real-life applications involving probability. When they conduct experiments such as throwing a die 1000 times which number do you expect to be on the upper face of the die among the numbers 1, 2,3,4,5 or 6?

Answers to Review Exercise on Unit 8

Help students to check their answer using SPSS or IT whenever necessary.

1. Data presentation is facilitating statistical analysis and illustrating the data using graphs and diagrams whereas, tabulation of Statistical Data is putting data into a statistical table.
2. a.

Amount of money (v)	1	2	3	4	5	6	7	8	9
Frequency (f)	1	4	2	3	2	2	1	1	4

   b. $\frac{\mathbf{7}}{\mathbf{20}} = \mathbf{3.5} = \mathbf{35}\%$
3. a. Sample space $= \{b_{1,} b_{2,} b_{3,} b_{4,} b_{5,} r_{1,} r_{2,} r_{3,} r_4\}$ where, b and r represent blue & red balls respectively.

   b. $\frac{4}{9}$
4. a. $\{H, T\}$, where H and T are head and tail of tossing a coin.

   b. $\{1, 2, 3, 4, 5, 6\}$, where each number is a face of the die.

   c. $\{(H, 1), (H, 2), (H, 3)(H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3)(T, 4), (T, 5), (T, 6)\}$

d. The event that A and B are $(\mathrm{H}, 1), (\mathrm{H}, 3)$ or $(\mathrm{H}, 5)$. Hence, the probability of the event

$\mathrm{A} \,\&\, \mathrm{B} = \frac{3}{12} = \frac{1}{4}$

Similarly, the event that A or B are

$(\mathrm{H}, 1), (\mathrm{H}, 2), (\mathrm{H}, 3), (\mathrm{H}, 4), (\mathrm{H}, 5), (\mathrm{H}, 6), (\mathrm{T}, 1), (\mathrm{T}, 3), (\mathrm{T}, 5)$. Hence, the probability of the event A or $\mathrm{B} = \frac{9}{12} = \frac{1}{4}$

5.

$\mathrm{Temperature}(x_i)$	$\mathrm{Temp} - \mathrm{Mean} = \mathrm{deviation} = x_i - \bar{x}$	$(x_i - \bar{x})^2$
18	$18 - 19.2 = -1.2$	1.44
22	$22 - 19.2 = 2.8$	7.84
19	$19 - 19.2 = -0.2$	0.04
25	$25 - 19.2 = 5.8$	33.64
12	$12 - 19.2 = -7.2$	51.84
$\bar{x} = \frac{96}{5} = 19.2$	$\sum d_i = 0$	$\sum {d_i}^2 = 94.80$

The median is 19 after putting in increasing order. Every value is the mode. Why? The range is $25 - 12 = 13$. The variance, $\delta = \frac{\sum {d_i}^2}{N} = \frac{94.8}{5} = 18.96$ & the standard deviation is $\sqrt{\delta} = \sqrt{18.96} = 4.35$

6. $\mathrm{Mean} = \frac{(11)(6)+(12)(7)+(13)(5)+(14)(7)+(15)(3)+(16)(2)}{6+7+5+7+3+2} = \frac{390}{30} = 13$

$\mathrm{Median} = \text{the middle value} = \frac{15^{th}\text{ value} + 16^{th}\text{ value}}{2} = \frac{13+13}{2} = 13$

Mode = the most frequent value $= 12$ and 14

Range = the difference between the largest and the smallest value $= 16 - 11 = 5$.

v	11	12	13	14	15	16
f	6	7	5	7	3	2

Solution:

$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{390}{30} = 13$

x_i	f_i	$x_i f_i$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
11	6	66	4	24
12	7	84	1	7
13	5	65	0	0
14	7	98	1	7
15	3	45	4	12
16	2	32	9	18
	$\sum f_i = 30$	$\sum x_i f_i = 390$	19	$\sum f_i (x_i - \bar{x})^2 = 68$

The variance $= \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i} = \frac{68}{30} \cong 2.267$

Standard deviation, $\delta = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}} \cong \sqrt{2.267} \cong 1.50554$

7. Which of the following is true?

a. False

b. False

c. False

d. True

8. The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. So, the probability that a 1, 4, 5, OR 6 will be on the upper face is $\frac{4}{6} = \frac{2}{3}$

9. **i.** the probability of spinning an 8 on the spinner if you know the arrow landed on an even number is $\frac{1}{2}$

ii. the probability of spinning a 9 on the spinner if you know the arrow landed on an even number is $\frac{1}{1} = 1$

10. The probability that the product of the numbers on the upper faces is:

Sample space =

$(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2) \ldots. (6,6)$

i. There is only one pair which is $(1,1)$ of numbers from the sample space.

So, the probability that the product of the numbers on the upper faces is 1 is $\frac{1}{36}$

ii. $\frac{10}{36} = \frac{5}{12}$

iii. $\frac{7}{36}$

iv. $\frac{13}{36}$

v. $\frac{1}{36}$

11. Probability of getting a queen of clubs $= \frac{1}{52}$

Probability of getting a king of hearts $= \frac{1}{52}$

Since only one card is drawn-getting both is an impossible outcome.

Hence, the probability of getting either a queen of clubs or a king of hearts is

$\frac{1}{52} + \frac{1}{52} = \frac{1}{26}$

12. B **13.** A

Trigonometric table

Corresponding angles (in degree)

	sin	cos	tan		
0	0	1	0		90
1	0.017452	0.999848	0.017455	57.2900	89
2	0.03490	0.999391	0.034921	28.63628	88
3	0.052336	0.99863	0.052408	19.08115	87
4	0.069756	0.997564	0.069927	14.30068	86
5	0.087156	0.996195	0.087489	11.43006	85
6	0.104528	0.994522	0.10510	9.514373	84
7	0.121869	0.992546	0.122784	8.144353	83
8	0.139173	0.990268	0.140541	7.115376	82
9	0.156434	0.987688	0.158384	6.313757	81
10	0.173648	0.984808	0.176327	5.671287	80
11	0.190809	0.981627	0.19438	5.144558	79
12	0.207912	0.978148	0.212556	4.704634	78
13	0.224951	0.97437	0.230868	4.33148	77
14	0.241922	0.97030	0.249328	4.010784	76
15	0.258819	0.965926	0.267949	3.732054	75
16	0.275637	0.961262	0.286745	3.487418	74
17	0.292371	0.95630	0.30573	3.270856	73
18	0.309017	0.951057	0.324919	3.077686	72
19	0.325568	0.945519	0.344327	2.904214	71
20	0.34202	0.939693	0.36397	2.74748	70
21	0.358368	0.933581	0.383864	2.605091	69
22	0.374606	0.927184	0.404026	2.475089	68
23	0.390731	0.920505	0.424474	2.355855	67
24	0.406736	0.913546	0.445228	2.246039	66
25	0.422618	0.906308	0.466307	2.144509	65
26	0.438371	0.898794	0.487732	2.050306	64
27	0.45399	0.891007	0.509525	1.962612	63
28	0.469471	0.882948	0.531709	1.880728	62
29	0.484809	0.87462	0.554308	1.80405	61
30	0.5000	0.866026	0.57735	1.732053	60
31	0.515038	0.857168	0.60086	1.664281	59
32	0.529919	0.848048	0.624869	1.600336	58
33	0.544639	0.838671	0.649407	1.539867	57
34	0.559192	0.829038	0.674508	1.482563	56
35	0.573576	0.819152	0.700207	1.42815	55
36	0.587785	0.809017	0.726542	1.376383	54
37	0.601815	0.798636	0.753553	1.327046	53
38	0.615661	0.788011	0.781285	1.279943	52
39	0.62932	0.777146	0.809783	1.23490	51
40	0.642787	0.766045	0.83910	1.191755	50
41	0.656059	0.75471	0.869286	1.15037	49
42	0.66913	0.743145	0.90040	1.110614	48
43	0.68200	0.731354	0.932514	1.07237	47
44	0.694658	0.71934	0.965688	1.035532	46
45	0.707106	0.707107	1.00000	1.00000	45
	cos	sin		tan	

Reference materials

You can use any teaching material related to the topic of discussion. You can get them from school library. If you have internet access, you can also search the appropriate topic and follow lecture notes and videos from Mathematics expertise located at different institutions. The following are some reference materials and important sites at this grade level.

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